Research and Innovation Center of Excellence

MSc on Intelligent Critical Infrastructure Systems

Machine Learning

Lecture 13: Monitoring and Control (Part II)

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funded by:

Course outline

- **Week 1**
  - Introduction and Preliminaries
- **Week 2**
  - Linear Regression
  - Regularisation, Logistic Regression, SVMs
- **Week 3**
  - Neural Networks and Deep Learning
- **Week 4**
  - Feature Engineering and Evaluation
  - Online Learning
- **Week 5**
  - Unsupervised Learning
- **Week 6**
  - Reinforcement Learning
- **Week 7**
  - **Monitoring and Control**

Application to Monitoring and Control – Outline

- Introduction to Monitoring and Control
- Mathematical Modelling of Dynamical Systems
- Adaptation and Learning in Control Systems
- System Identification (Continuous-time & Discrete-time)
- Learning Control using Neural Networks

→ CONCLUDING REMARKS FOR THE CLASS

Nonlinear Identification (Learning)

$$\dot{x}(t) = \xi(x(t), u(t), t) + f(x(t), u(t), t)$$
$$y(t) = \zeta(x(t), u(t), t) + h(x(t), u(t), t)$$

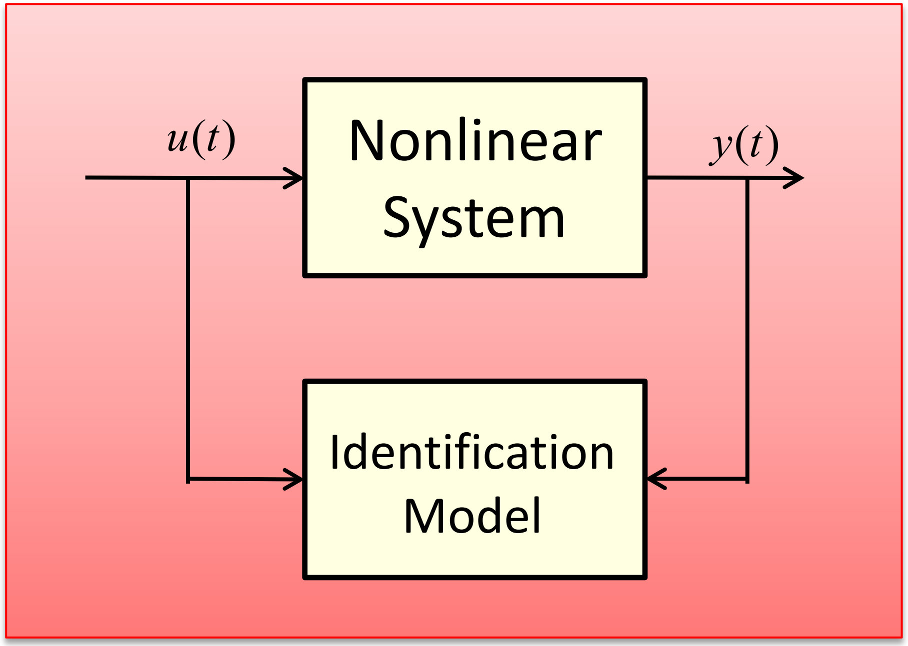
$x(t)$: **State variable**

$u(t)$: **Control input**

ξ, ζ : **Known parts**

f, h : **Unknown parts**

<u>Problem:</u> design an identification model that allows estimation of the unknown f and h.



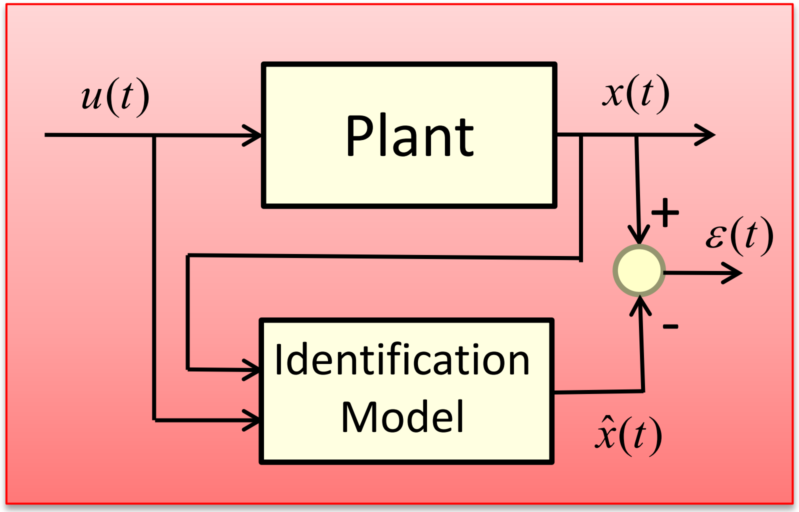
Start with a first-order linear example

$$\dot{x}(t) = -ax(t) + bu(t) \qquad a, b : \text{unknown}$$ **[Plant]**

$$\boxed{\dot{\hat{x}}(t) = -a_m \hat{x}(t) + \left(a_m - \hat{a}(t)\right) x(t) + \hat{b}(t) u(t)} \qquad a_m > 0$$ **[Estimation Scheme]**

$$\boxed{\begin{aligned} \dot{\hat{a}}(t) &= -\gamma_1 \varepsilon(t) x(t) \\ \dot{\hat{b}}(t) &= \gamma_2 \varepsilon(t) u(t) \end{aligned}}$$ **[Adaptive Laws]**

$$\varepsilon(t) = x(t) - \hat{x}(t)$$ **[Estimation Error]**



Design Parameters: $a_m > 0, \quad \gamma_1 > 0, \quad \gamma_2 > 0, \quad \hat{x}(0)$

Nonlinear Systems with Unknown Parameters

$$\dot{x}(t) = -ax(t) + cf(x(t)) + bg(u(t))$$

a, b, c : unknown

$f(\bullet), g(\bullet)$: known

$$\dot{\hat{x}}(t) = -a_m\hat{x}(t) + \left(a_m - \hat{a}(t)\right)x(t) + \hat{c}(t)f(x(t)) + \hat{b}(t)g(u(t))$$

Estimation Scheme

$$\hat{x}(t) = \frac{1}{s+a_m}\left[\left(a_m - \hat{a}(t)\right)x(t) + \hat{c}(t)f(x(t)) + \hat{b}(t)g(u(t))\right]$$

Adaptive Laws

$$\dot{\hat{a}}(t) = -\gamma_1\varepsilon(t)x(t)$$

$$\dot{\hat{b}}(t) = \gamma_2\varepsilon(t)g(u(t))$$

$$\dot{\hat{c}}(t) = \gamma_3\varepsilon(t)f(x(t))$$

$$\varepsilon(t) = x(t) - \hat{x}(t)$$

[Estimation Error]

Nonlinear Systems with Unknown Functions

$$\dot{x}(t) = \underbrace{\xi(x(t),u(t))}_{\text{known}} + \underbrace{f(x(t),u(t))}_{\text{unknown}}$$

Plant

$$\dot{\hat{x}}(t) = -a_m \hat{x}(t) + \xi(x(t),u(t)) + a_m x(t) + \hat{f}(x(t),u(t);\hat{\theta}(t))$$

Estimation Scheme

$$\hat{x}(t) = \frac{1}{s+a_m}\left[\xi(x(t),u(t)) + a_m x(t) + \hat{f}(x(t),u(t);\hat{\theta}(t))\right]$$

Adaptive Laws

$$\dot{\hat{\theta}} = \Gamma Z^T \varepsilon$$

$$Z = \frac{\partial \hat{f}}{\partial \hat{\theta}}(x,u,\hat{\theta})$$

$$\varepsilon = x - \hat{x}$$

$$\Gamma > 0 \qquad \text{(positive definite)}$$

Continuous-Time Learning Using Neural Networks

$$\dot{x}(t) = f(x(t)) + u(t) \qquad f: \text{unknown}$$

$$\dot{\hat{x}}(t) = -a_m \hat{x}(t) + a_m x(t) + \hat{f}(x(t); \hat{\theta}(t)) + u(t) \qquad a_m > 0$$

$$\dot{\hat{\theta}}(t) = \Gamma Z(t)^T \varepsilon(t)$$

A. Radial Basis Function (RBF) Networks

$$\hat{f}(x; \hat{\theta}_1, \hat{\theta}_2, \ldots \hat{\theta}_N) = \sum_{i=1}^{N} \hat{\theta}_i e^{-(x-c_i)^2/\sigma^2}$$

$$Z_i = \frac{\partial \hat{f}}{\partial \hat{\theta}_i} = e^{-(x-c_i)^2/\sigma^2}$$

$$\dot{\hat{\theta}}_i = \gamma_i e^{-(x-c_i)^2/\sigma^2} \left(x - \hat{x}\right)$$

Continuous-Time Learning Using Neural Networks

Implementation using RBF Networks

$$\dot{x} = f(x) + u \qquad x(0) = x^0$$

$$\dot{\hat{x}} = -a_m \hat{x} + a_m x + \sum_{i=1}^{N} \left(\hat{\theta}_i e^{-(x-c_i)^2/\sigma^2} \right) + u \qquad \hat{x}(0) = \hat{x}^0$$

$$\dot{\hat{\theta}}_i = \gamma_i e^{-(x-c_i)^2/\sigma^2} (x - \hat{x}) \qquad \hat{\theta}_i(0) = \hat{\theta}_i^0$$

Design parameters to choose:

$a_m, \sigma, \hat{x}^0, \gamma_i, c_i, \hat{\theta}_i^0 \qquad i = 1, 2, \ldots N$

Continuous-Time Learning Using Neural Networks

B. Sigmoidal Neural Networks (SNN)

$$\hat{f}(x;\hat{\theta}_1,\hat{\theta}_2,\ldots\hat{\theta}_{3M}) = \sum_{i=1}^{M} \hat{\theta}_i \sigma\left(\hat{\theta}_{i+M} x + \hat{\theta}_{i+2M}\right)$$

$$\sigma(p) = \frac{1}{1+e^{-p}} \quad \Rightarrow \quad \sigma(\alpha x + \beta) = \frac{1}{1+e^{-(\alpha x+\beta)}}$$

$$Z_i = \frac{\partial \hat{f}}{\partial \hat{\theta}_i}$$

$$\dot{\hat{\theta}}_i = \gamma_i Z_i \left(x - \hat{x}\right)$$

Continuous-Time Learning Using Neural Networks

Implementation of SNN

$$\dot{x} = f(x) + u \qquad x(0) = x^0$$

$$\dot{\hat{x}} = -a_m \hat{x} + a_m x + \sum_{i=1}^{M} \hat{\theta}_i \sigma\left(\hat{\theta}_{i+M} x + \hat{\theta}_{i+2M}\right) + u \qquad \hat{x}(0) = \hat{x}^0$$

$$\dot{\hat{\theta}}_i = \gamma_i \frac{\partial \hat{f}}{\partial \hat{\theta}_i}(x - \hat{x}) \qquad \hat{\theta}_i(0) = \hat{\theta}_i^0$$

Design parameters to choose:

$a_m, \hat{x}^0, \gamma_i, \hat{\theta}_i^0 \qquad i = 1, 2, \ldots 3M$

Discrete-time Learning in dynamical systems

$$y(k+1) = f\left(y(k), y(k-1), \ldots y(k-n_y, u(k), \ldots u(k-n_u\right)$$

$k \in \mathbb{Z}^+ \qquad k = 0, 1, 2, \ldots\ldots$

$f : \mathbb{R}^{n_y+1} \times \mathbb{R}^{n_u+1} \to \mathbb{R}$

f is unknown (or partially unknown)

Let $z(k) = \left[y(k), y(k-1), \ldots y(k-n_y, u(k), \ldots u(k-n_u\right]$

$\Rightarrow \boxed{y(k+1) = f(z(k))} \qquad (z(k)$ is measurable)

Discrete-Time Learning

$$y(k+1) = f(z(k))$$
$$\hat{y}(k+1) = \hat{f}(z(k);\hat{\theta}(k))$$

Prediction/Identification Model

Adaptive Laws:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \alpha_0 e(k+1)\xi(k)$$

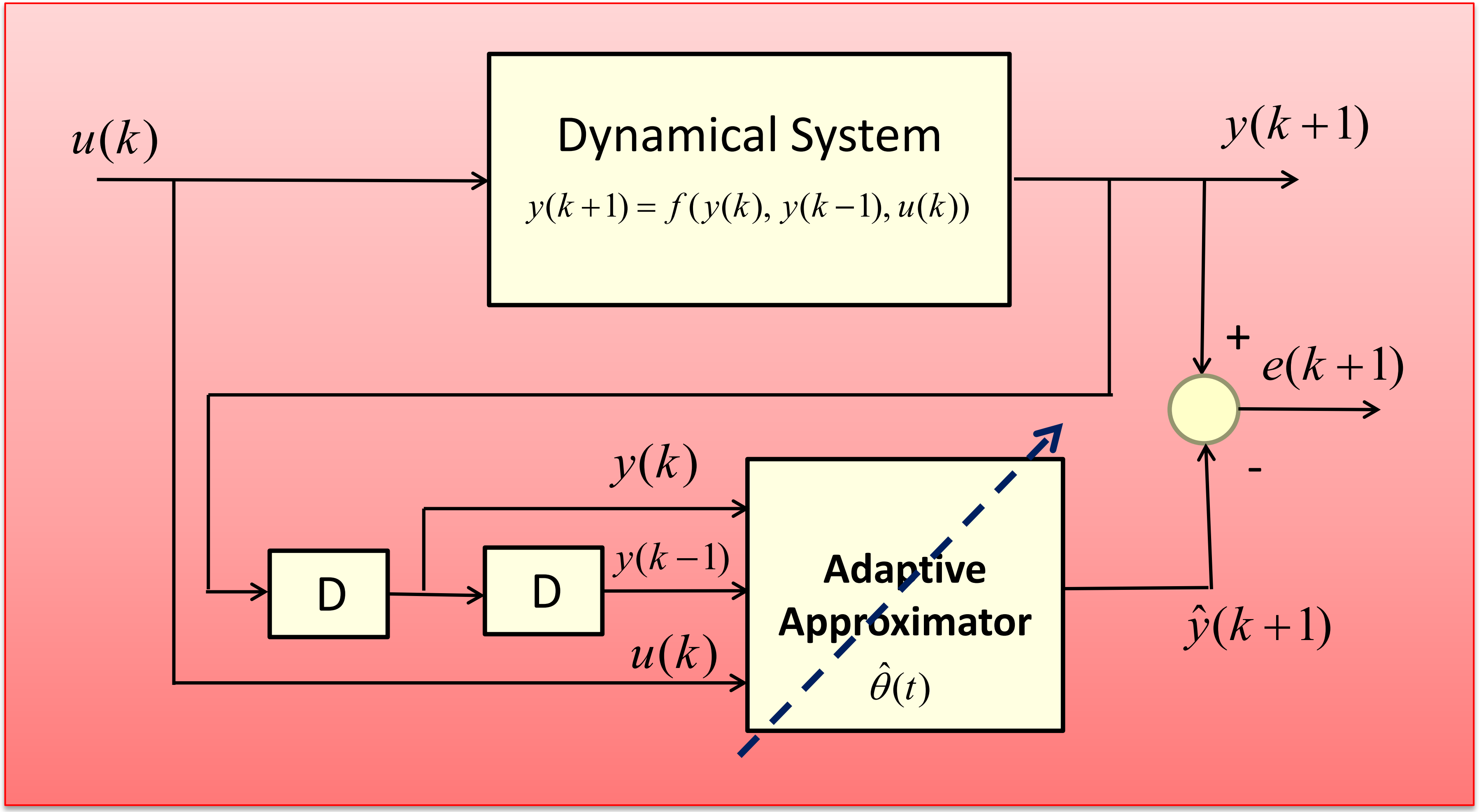
$$\hat{\theta}(k+1) = \hat{\theta}(k) + \frac{\gamma_0 e(k+1)}{\beta_0 + |\xi(k)|^2}\xi(k)$$

Normalized Gradient Descent

$\alpha_0 > 0$	Step size
$0 < \gamma_0 < 2$	Learning rate
$\beta_0 > 0$	Small design constant
$\xi(k) = \frac{\partial \hat{f}}{\partial \hat{\theta}}\left(z(k),\hat{\theta}(k)\right)$	Network sensitivity function
$e(k) = y(k) - \hat{y}(k)$	Estimation error

Discrete-Time Learning

Example: $y(k+1) = f(y(k), y(k-1), u(k))$



Discrete-Time Learning

Example: $y(k+1) = f(y(k), y(k-1), u(k))$

Given $y(0), y(-1), \hat{\theta}(0), u(0)$

\begin {for} $k = 0, 1, 2, \ldots\ldots N$

1. $z(k) = \left[y(k), y(k-1), u(k) \right]$

2. $\xi(k) = \dfrac{\partial \hat{f}}{\partial \hat{\theta}} \left(z(k), \hat{\theta}(k) \right)$

3. $y(k+1) = f(z(k))$

4. $\hat{y}(k+1) = \hat{f}(z(k), \hat{\theta}(k))$

5. $\hat{\theta}(k+1) = \hat{\theta}(k) + \dfrac{\gamma_0 \left(y(k+1) - \hat{y}(k+1) \right)}{\beta_0 + \left| \xi(k) \right|^2} \xi(k)$

\end {for}

From Learning to Fault Diagnosis

- A similar estimation scheme can also be used for fault diagnosis
- Additionally, in fault diagnosis we require upper bounds (or statistical information) on the uncertainty so that we can distinguish between faults and modelling uncertainties
- Based on the modelling uncertainty bounds, we can compute adaptive threshold signals
- Finally, we also need to design a decision logic algorithm for fault detection and isolation

Learning Control Using Neural Networks

$$\dot{x}(t) = \xi(x(t), u(t), t) + f(x(t), u(t), t)$$
$$y(t) = \zeta(x(t), u(t), t) + h(x(t), u(t), t)$$

$x(t):$ **State variable**

$u(t):$ **Control input**

$\xi,\ \zeta:$ **Known parts**

$f,\ h:$ **Unknown parts**

Problem: design a controller:

$$u = \mu(y, \theta, t)$$
$$\dot{\theta} = \lambda(u, y, \theta, t)$$

Such that y(t) follows a desired trajectory $y_r(t)$ as closely as possible.

Start with a simple case: linear adaptive control

$$\dot{y}(t) = ay(t) + bu(t)$$

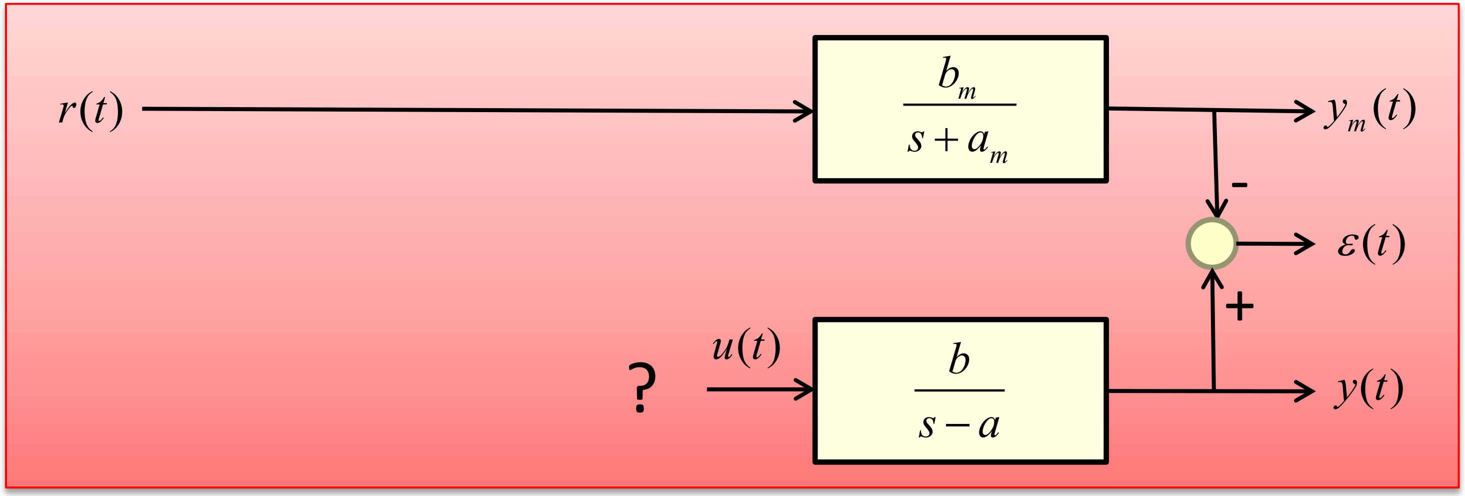
a, b: unknown; sign(b) is known

<u>Model Reference Adaptive Control (MRAC) Problem:</u>

Choose an appropriate control law u(t) such that all signals in the closed loop plant are bounded and the plant output y(t) tracks the output $y_m(t)$ of the reference model:

$$\dot{y}_m(t) = -a_m y_m(t) + b_m r(t)$$

where r(t) is a bounded, piecewise continuous signal, referred to as the reference or command input.



Start with a simple case: linear adaptive control

How would we solve the MRAC problem if we knew a, b?

$$u = -k^* y(t) + l^* r(t)$$

$$\Rightarrow \quad k^* = \frac{a_m + a}{b} \qquad l^* = \frac{b_m}{b}$$

Assuming that b is not equal to 0 (plant is controllable)

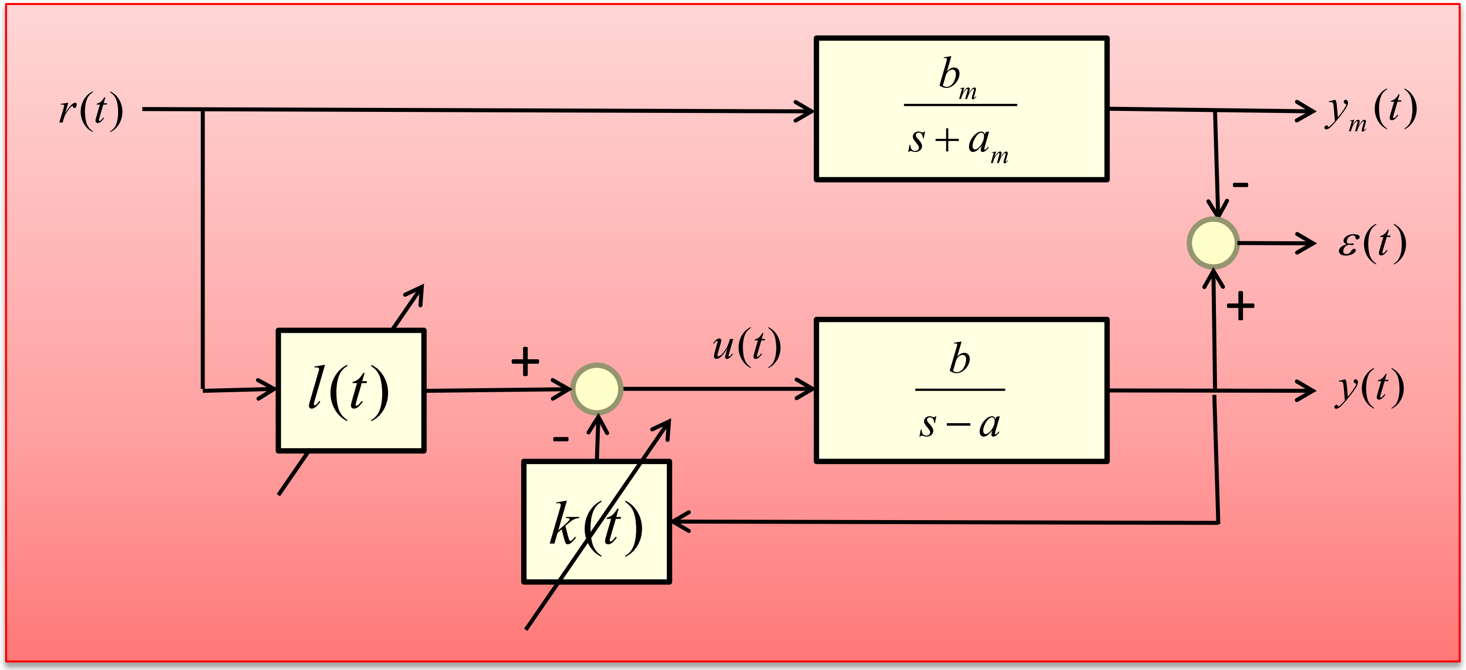
$$\frac{y(s)}{r(s)} = \frac{y_m(s)}{r(s)} \quad \Rightarrow \quad \left| y(t) - y_m(t) \right| \to 0 \ \ \text{(exponentially fast for all r(t))}$$

Direct Adaptive Control (Direct MRAC)

$$\dot{y}(t) = ay(t) + bu(t)$$
$$\dot{y}_m(t) = -a_m y_m(t) + b_m r(t)$$
$$u(t) = -k(t)y(t) + l(t)r(t)$$
$$\varepsilon(t) = y(t) - y_m(t)$$

$$\dot{k}(t) = \gamma_1 \varepsilon(t) y(t) \operatorname{sgn}(b)$$
$$\dot{l}(t) = -\gamma_2 \varepsilon(t) r(t) \operatorname{sgn}(b)$$

$$\dot{y}(t) = (a - bk(t))y(t) + bl(t)r(t)$$
$$\dot{y}_m(t) = -a_m y_m(t) + b_m r(t)$$



Indirect Adaptive Control (Indirect MRAC)

If a, b were known, then we would use:

$$u = -k^* y(t) + l^* r(t)$$

$$\begin{cases} k^* = \dfrac{a_m + a}{b} \\ l^* = \dfrac{b_m}{b} \end{cases}$$

Replace the unknown a, b by their estimates $\hat{a}(t), \hat{b}(t)$ where they are generated by identification techniques:

$$u(t) = -\frac{a_m + \hat{a}(t)}{\hat{b}(t)} y(t) + \frac{b_m}{\hat{b}(t)} r(t) \qquad \text{(IAC)}$$

$$u(t) = -k(t) y(t) + l(t) r(t) \qquad \text{(DAC)}$$

Need to make sure that: $\hat{b}(t) \neq 0$ (otherwise $u(t) \to \infty$)

Indirect Adaptive Control (Indirect MRAC)

Identification Model (Estimation Scheme):

$$\dot{\hat{y}}(t) = -a_m \hat{y}(t) + (a_m + \hat{a}(t))y(t) + \hat{b}(t)u(t)$$

Identification Error:

$$\varepsilon_i(t) = y(t) - \hat{y}(t)$$

Adaptive Laws:

$$\dot{\hat{a}}(t) = \gamma_1 \varepsilon_i(t) y(t)$$

$$\dot{\hat{b}}(t) = \gamma_2 \varepsilon_i(t) u(t)$$

Indirect Adaptive Control (Indirect MRAC)

How do we guarantee that: $\hat{b}(t) \neq 0$

Instead of: $\dot{\hat{b}}(t) = \gamma_2 \varepsilon_i(t) u(t)$

Use:

$$\dot{\hat{b}}(t) = \begin{cases} \gamma_2 \varepsilon_i(t) u(t) & \text{if } \hat{b} > \bar{b} \\ \gamma_2 \varepsilon_i(t) u(t) & \text{if } \hat{b} = \bar{b} \text{ and } \gamma_2 \varepsilon_i u > 0 \\ 0 & \text{if } \hat{b} = \bar{b} \text{ and } \gamma_2 \varepsilon_i u \leq 0 \end{cases}$$

This will guarantee that $\hat{b}(t) \geq \bar{b}$ and also $u \in L_\infty$

Need to assume that we know the sign of b and also that we know a lower bound on b.

Nonlinear Systems (with unknown functions)

$$\dot{y} = f(y) + u$$

Objective: Choose an appropriate control law $u(t)$ such that all the signals in the closed-loop plant are bounded and the plant output $y(t)$ tracks the output $y_m(t)$ of the reference model

$$\dot{y}_m = -a_m y_m + b_m r$$

for all $r(t)$ that are bounded and piecewise continuous.

Nonlinear Systems (with unknown functions)

If we knew f(y) then what would we do?

$$u = -f(y) - a_m y + b_m r$$

$$\dot{y} = f(y) - f(y) - a_m y + b_m r$$

$$\dot{y} = -a_m y + b_m r$$

$$\dot{y}_m = -a_m y_m + b_m r$$

$$\lim_{t \to \infty} \left\| y(t) - y_m(t) \right\| = 0$$

Exponentially fast

Learning Control with Neural Networks (indirect approach)

$$u = -\hat{f}(y,\hat{\theta}) - a_m y + b_m r$$

Identification Model (Estimation Scheme):

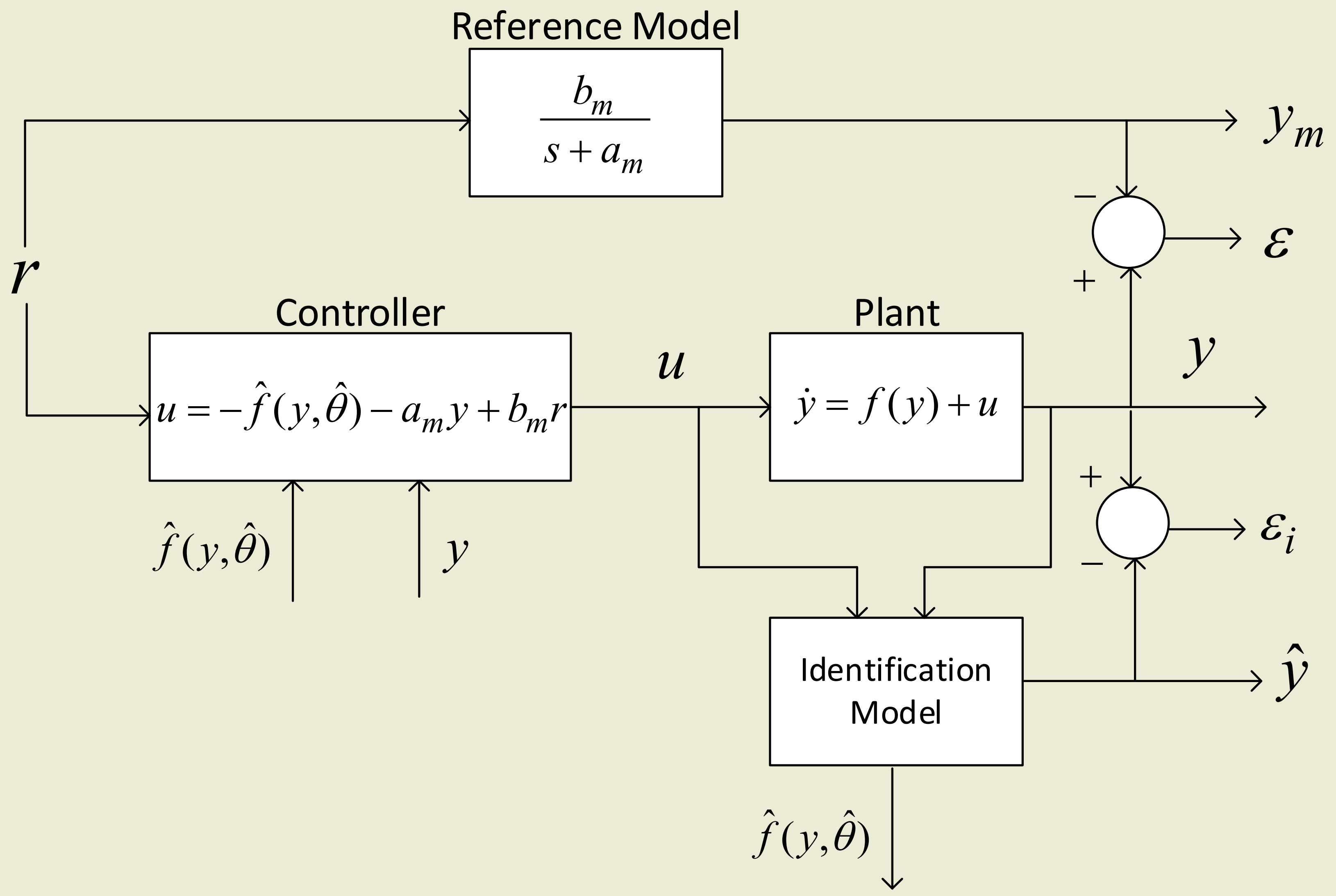
$$\dot{\hat{y}} = -a_m \hat{y} + a_m y + \hat{f}(y,\hat{\theta}) + u$$

$$\dot{\hat{\theta}} = \Gamma Z^{\top} e_i$$

$$e_i = y - \hat{y}$$

$$Z = \frac{\partial \hat{f}(\hat{y},\hat{\theta})}{\partial \hat{\theta}}$$

Learning Control with Neural Networks (indirect approach)



Learning Control with Neural Networks (indirect approach)

$$\dot{y} = f(y) \underbrace{- \hat{f}(y,\hat{\theta}) - a_m y + b_m r}_{u}, \qquad y(0) = y_o$$

$$\dot{\hat{y}} = -a_m \hat{y} + a_m y + \hat{f}(y,\hat{\theta}) + u, \qquad \hat{y}(0) = \hat{y}_o$$

$$\dot{y}_m = -a_m y + b_m r, \qquad y_m(0) = y_m^o$$

$$\dot{\hat{\theta}} = \Gamma \frac{\partial \hat{f}}{\partial \hat{\theta}} \underbrace{(y - \hat{y})}_{e_i}, \qquad \hat{\theta}(0) = \hat{\theta}^o$$

Discrete-time Learning Control

$$y(k+1) = f\left(y(k), y(k-1), \ldots y(k-n_y)\right) + u(k)$$

Objective: Choose an appropriate control law u(k) such that all the signals in the closed-loop plant are bounded and the plant output y(k) tracks the output $y_m(k)$ of the reference model $y_m(k+1) = -a_m y_m(k) + b_m r(k)$ for all r(k) that are bounded.

f is unknown (or partially unknown)

Let $z(k) = \left[y(k), y(k-1), \ldots y(k-n_y)\right]$

$\Rightarrow y(k+1) = f(z(k)) + u(k)$ ($z(k)$ is measurable)

Discrete-Time Learning Control

$$y(k+1) = f(z(k)) + u(k)$$

$$\hat{y}(k+1) = \hat{f}(z(k);\hat{\theta}(k)) + u(k)$$ **Prediction/Identification Model**

$$u(k) = -\hat{f}(z(k);\hat{\theta}(k)) - a_m y_m(k) + b_m r(k)$$ **Learning Control Law**

Adaptive Laws:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \alpha_0 e(k+1)\xi(k)$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \frac{\gamma_0 e(k+1)}{\beta_0 + |\xi(k)|^2}\xi(k)$$

Normalized Gradient Descent

$\alpha_0 > 0$ Step size

$0 < \gamma_0 < 2$ Learning rate

$\beta_0 > 0$ Small design constant

$\xi(k) = \frac{\partial \hat{f}}{\partial \hat{\theta}}\left(z(k),\hat{\theta}(k)\right)$ Network sensitivity function

$e(k) = y(k) - \hat{y}(k)$ Estimation error

Discrete-Time Learning Control

Example: $y(k+1) = f(y(k),\, y(k-1)) + u(k)$

Given $\quad y(0),\, y(-1),\, \hat{\theta}(0),\, y_m(0)$

\begin {for} $\quad k = 0, 1, 2, \ldots\ldots N$

1. $z(k) = \left[y(k),\, y(k-1)\right]$
2. $u(k) = -\hat{f}(z(k), \hat{\theta}(k)) - a_m y_m(k) + b_m r(k)$
3. $\xi(k) = \dfrac{\partial \hat{f}}{\partial \hat{\theta}}\left(z(k), \hat{\theta}(k)\right)$
4. $y(k+1) = f(z(k)) + u(k)$
5. $y_m(k+1) = -a_m y_m(k) + b_m r(k)$
6. $\hat{y}(k+1) = \hat{f}(z(k), \hat{\theta}(k))$
7. $\hat{\theta}(k+1) = \hat{\theta}(k) + \dfrac{\gamma_0\left(y(k+1) - \hat{y}(k+1)\right)}{\beta_0 + \left|\xi(k)\right|^2}\xi(k)$

\end {for}

Some Bibliography on Monitoring and Control Using Learning Methods

- J.A. Farrell and M.M. Polycarpou, "*Adaptive Approximation Based Control*", J. Wiley, 2006.
- M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. Diagnosis and Fault-Tolerant Control. Springer-Verlag, 2016.

Some Final Conclusions

- We have learned a lot
- A lot that we have not learned!
- The importance of gaining intuition
- Never lose the big picture and the applications