Optimization of CIS

ECE 802

Tutorial 4 – Optimization Problems

L. Tziovani and A. Astolfi

Problem A: The assignment problem

This is the problem of assigning n people to n jobs so as to maximize some overall level of competence. For example, person i might take an average time $t_{i,j}$ to do job j. The objective is to assign each person to a job by minimizing the total time for all tasks. Formulate the considered optimization problem.

For the problem formulation, use the binary variable $x_{i,j}$ to indicate if person i is assigned to job j ($x_{i,j} = 1$).

<u>Solution</u>:

$$\text{Minimize} \qquad \sum_i \sum_j t_{i,j}\, x_{i,j}$$

$$\text{s.t.} \quad \sum_i x_{i,j} = 1 \quad \forall j \quad (1)$$

$$\sum_j x_{i,j} = 1 \quad \forall i \quad (2)$$

Constraint (1) impose the condition that each job be filled. Constraints (2) impose the condition that every person be assigned a job.

Problem B: The blending problem

A food is manufactured by refining raw oils and blending them together. The raw oils come in

Vegetable oils	VEG 1
Vegetable oils	VEG 2
Non-vegetable oils	OIL 1
Non-vegetable oils	OIL 2
Non-vegetable oils	OIL 3

Vegetable oils and non-vegetable oils require different production lines for refining. In any month it is not possible to refine more than 200 tons of vegetable oil and more than 250 tons of non-vegetable oils. There is no loss of weight in the refining process and the cost of refining may be ignored.

There is a technological restriction of hardness in the final product. In the units in which hardness is measured, this must lie between 3 and 6. It is assumed that hardness blends linearly. The costs (per ton) and hardness of the raw oil

	VEG 1	VEG 2	OIL 1	OIL 2	OIL 3
Cost (€)	110	120	130	110	115
Hardness	8.8	6.1	2.0	4.2	5.0

The final product sells for 150 per ton. How should the food manufacturer make their product in order to maximize their net profit? Formulate the considered optimization problem and solve it using an optimization solver.

Solution:

Objective function:

$$max\ \ f = 40x_1 + 30x_2 + 20x_3 + 40x_4 + 35x_5$$

subject to:

$$x_1 + x_2 \leq 200$$

$$x_3 + x_4 + x_5 \leq 250$$

$$8.8x_1 + 6.1x_2 + 2x_3 + 4.2x_4 + 5x_5 \leq 6(x_1 + x_2 + x_3 + x_4 + x_5)$$

$$8.8x_1 + 6.1x_2 + 2x_3 + 4.2x_4 + 5x_5 \geq 3(x_1 + x_2 + x_3 + x_4 + x_5)$$

or

$$x_1 + x_2 \leq 200$$

$$x_3 + x_4 + x_5 \leq 250$$

$$2.8x_1 + 0.1x_2 - 4x_3 - 1.8x_4 - x_5 \leq 0$$

$$5.8x_1 + 3.1x_2 - 1x_3 + 1.2x_4 + 2x_5 \geq 0$$

Solution: x=[159.25, 40.74, 0, 250, 0], net profit = 17592

Problem C: Unit Commitment

Consider a power system with eight units which have the following quadratic cost functions

1 GT unit: C(P) = 710 + 60P + 0.37P^2 $€/\mathrm{h}$, $5 \le P \le 30\ MW$

3 ST units: C(P) = 670 + 40P + 0.4P^2 $€/\mathrm{h}$, $25 \le P \le 70\ MW$

2 ICE units: C(P) = 150 + 38P + 0.28P^2 $€/\mathrm{h}$, $7 \le P \le 28\ MW$

2 ST units: C(P) = 870 + 37P + 0.18P^2 $€/\mathrm{h}$, $50 \le P \le 140\ MW$

This problem schedules the conventional generating units to ensure the power balance between generation and demand by minimizing the total generation cost. Specifically, this problem defines which units will be committed and finds their produced power P.

Solve the problem for (a) load demand= 418 MW and (b) load demand= 86.9 MW.

For the problem formulation, use the binary variable z_i to indicate if the unit i is *ON* ($z_i = 1$) or *OFF* ($z_i = 0$). When a unit is OFF, then its produced power is zero ($P_i = 0$).

<ins>Solution</ins>:

$$\text{Minimize} \qquad \sum_{i=1}^{I} a_i z_i + b_i P_i + c_i P_i^2$$

$$\text{s.t.} \quad z_i \underline{P_i} \le \ P_i \le z_i \bar{P}_i, \qquad \forall i \in I \quad (1)$$

$$\sum_{i=1}^{I} P_i = L \quad (2)$$

where a_i, b_i, and c_i denote the cost coefficients of each quadratic cost function $i \in I$ ($I = \{1..8\}$), $\underline{P_i}$ and $\bar{P}_i$ the minimum and maximum generating power of unit i, and L the load demand.