ECE 802 - Optimization of CIS

A. Astolfi and L. Tziovani

COURSE-WORK

The final report of the Part I should be submitted by the 14th of October while the report of Part II should be submitted by the 31th of October 2022. Both reports and Matlab codes should be submitted electronically to ltziov01@ucy.ac.cy

Part I - Unconstrained Optimization [60 marks]

To compare the performance of optimisation methods it is customary to construct *test* functions and then compare the behavior of various optimization algorithms for such functions.
One, often used, test function is the so-called Rosenbrock function, *i.e.*

$$v(x, y) = 100(y - x^2)^2 + (1 - x)^2.$$

A1) Compute analytically all stationary points of the function $v(x, y)$ and verify if they are minimizers/maximizers/saddle points. [3 marks]

A2) Plot (using Matlab or a similar SW) the level sets of the function $v(x, y)$. [3 marks]

A3) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the gradient method (see Section 2.5) with Armijo line search (see Section 2.4.2). [7 marks]

A4) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using Newton method (see Section 2.6) with and without Armijo line search. [7 marks]

A5) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Polak-Ribiere algorithm (see Section 2.7.3) with Armijo line search. [9 marks]

A6) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Broyden-Fletcher-Goldfarb-Shanno algorithm (see Section 2.8) with Armijo line search. [9 marks]

A7) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the simplex method (see Section 2.9). Consider modifying the method in Section 2.9 to improve convergence properties of the algorithm. [10 marks]

A8) *Run* the minimization procedures written in points A3) to A7) with initial point $(x_0, y_0) = (-3/4, 1)$.

A8a) Plot, on the (x, y)-plane, the sequences of points generated by each algorithm. Are these sequences converging to a stationary point of $v(x, y)$? [3 marks]

A8b) For a sequence $\{x_k, y_k\}$ consider the *cost*

$$J_k = \log\left((x_k - 1)^2 + (y_k - 1)^2\right).$$

Plot, for each of the sequences generated by the above algorithms, the cost J_k as a function of k. Use such a plot to assess the speed of convergence of each of the considered algorithms. [9 marks]

Part II - Constrained Optimization [40 marks]

B1) **Assignment Problem**: Consider the problem of assigning agents to do $\mathcal{N} = \{1, ..., N\}$ tasks, where $\mathcal{M} = \{1, ..., M\}$ agents, $M \geq N$, are available to be assigned to any task. All tasks must be completed and (a) only one agent can be assigned to a task, and (b) an agent can do only one task. Note that some agents will not be assigned to any task. Assuming that $T_{m,n}$ denotes the time where agent $m \in M$ needs to execute task $n \in N$, assign the agents to the tasks by minimizing the total time of execution.
A) Formulate mathematically the problem as an Integer Program. [7 marks]
B) Solve the problem considering the execution times given in Table 1, where $N = 5$ and $M = 8$. [7 marks]
C) Reformulate the aforementioned problem considering the constraint that if agent $m = 1$ is assigned to a task, then agent $m = 2$ cannot be assigned to any task. [3 marks]
D) Solve Problem B1.C considering the execution times given in Table 1. [3 marks]

Table 1: Execution times $T_{m,n}$

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	3.2	4.6	8	2.4	5
$m=2$	1.5	5	4	4.2	5.1
$m=3$	8.8	6.1	4.5	4.2	5.9
$m=4$	2.8	3.5	3.9	5.4	6.5
$m=5$	4.8	5.1	3.5	4.9	5.5
$m=6$	8.1	4.1	3.2	7.2	7.0
$m=7$	3.3	3.6	4.1	5.5	5.4
$m=8$	4.4	7.1	4.6	4.3	5.5

B2) **Unit Commitment (UC)**: Consider a power system with four generating units, $\mathcal{I} = \{1, 2, 3, 4\}$, which have the following cost functions and power limits ($\underline{P}_i$, $\overline{P}_i$)

$$\text{Unit 1: } C_1(P_1) = 800 + 39P_1 \text{ €/h}, \quad 5 \leq P_1 \leq 30 \text{ MW}$$

$$\text{Unit 2: } C_2(P_2) = 670 + 40P_2 \text{ €/h}, \quad 25 \leq P_2 \leq 70 \text{ MW}$$

$$\text{Unit 3: } C_3(P_3) = 150 + 38P_3 \text{ €/h}, \quad 7 \leq P_3 \leq 28 \text{ MW}$$

$$\text{Unit 4: } C_4(P_4) = 700 + 37P_4 \text{ €/h}, \quad 50 \leq P_4 \leq 140 \text{ MW}$$

The UC problem schedules the conventional generating units to ensure the power balance between generation and demand by minimizing the total generation cost. Specifically, this problem defines which units will be committed and finds their produced power P_i, $i \in I$, by satisfying the constraint that the total produced power is equal to the load demand L.
A) Formulate mathematically the problem as a mixed-integer linear program (MILP). [7 marks]
B) Solve the problem for $L = 190$ MW. [7 marks]
C) Reformulate the aforementioned problem considering the spinning reserve constraint which ensures that an available amount of extra power (available reserve) can be generated from all the committed units, by increasing their generation, in case of variability of the load demand in real operation. Let $\overline{P}_i$ denote the maximum power of unit i and R_i the

available reserve of each unit, defined as $R_i = \overline{P}_i - P_i$ when the unit i is *ON* and $R_i = 0$ when the unit is *OFF*. The reserve constraint ensures that the total available reserve from all units is equal or greater than the minimum spinning reserve $\overline{R}$. [3 marks]
D) Solve Problem B2.C for $\overline{R} = 40$ MW. [3 marks]

Note: For the problem formulation, use the binary variable z_i to indicate if the unit $i \in I$ is *ON* (committed), $z_i = 1$, or *OFF*, $z_i = 0$. When a unit is *OFF*, then its produced power is zero, $P_i = 0$. This can be achieved using the constraint

$$z_i \underline{P}_i \leq P_i \leq z_i \overline{P}_i, \quad \forall i \in I. \tag{1}$$

Code Problems B1 and B2 in Matlab and solve them using the Gurobi solver or the intlinprog solver of Matlab.

Gurobi optimizer guide (pages 6-10)
`https://www.gurobi.com/wp-content/plugins/hd_documentations/documentation/9.0/quickstart_windows.pdf`