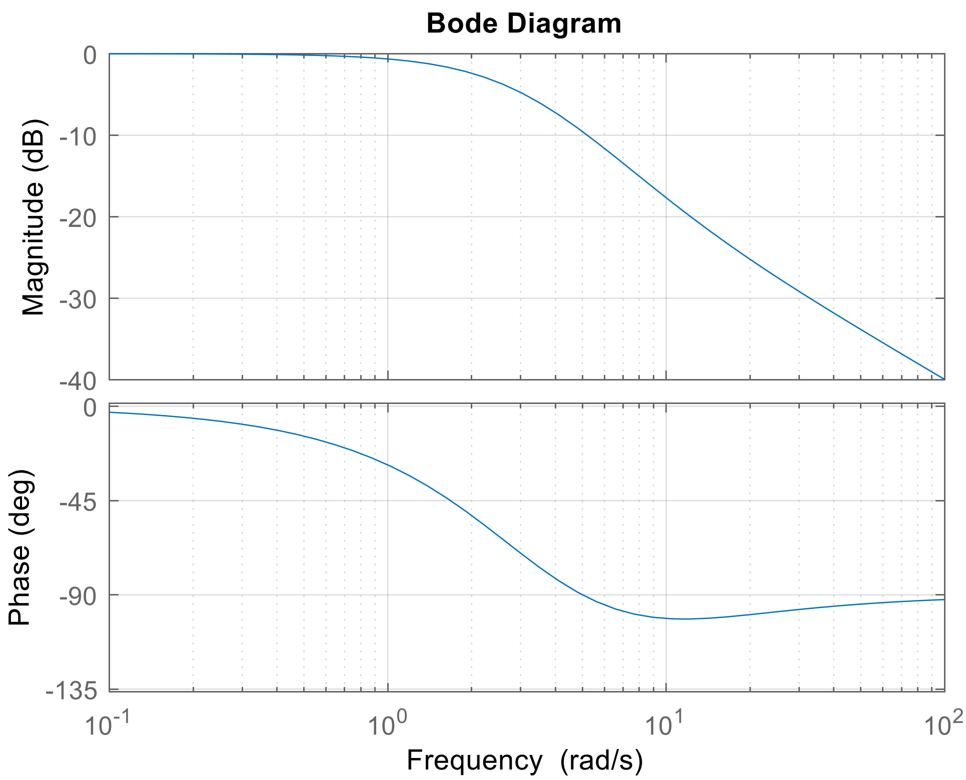
Tutorial 6

ECE 804-Bode Diagrams

Draw the Bode diagram of the following transfer functions:

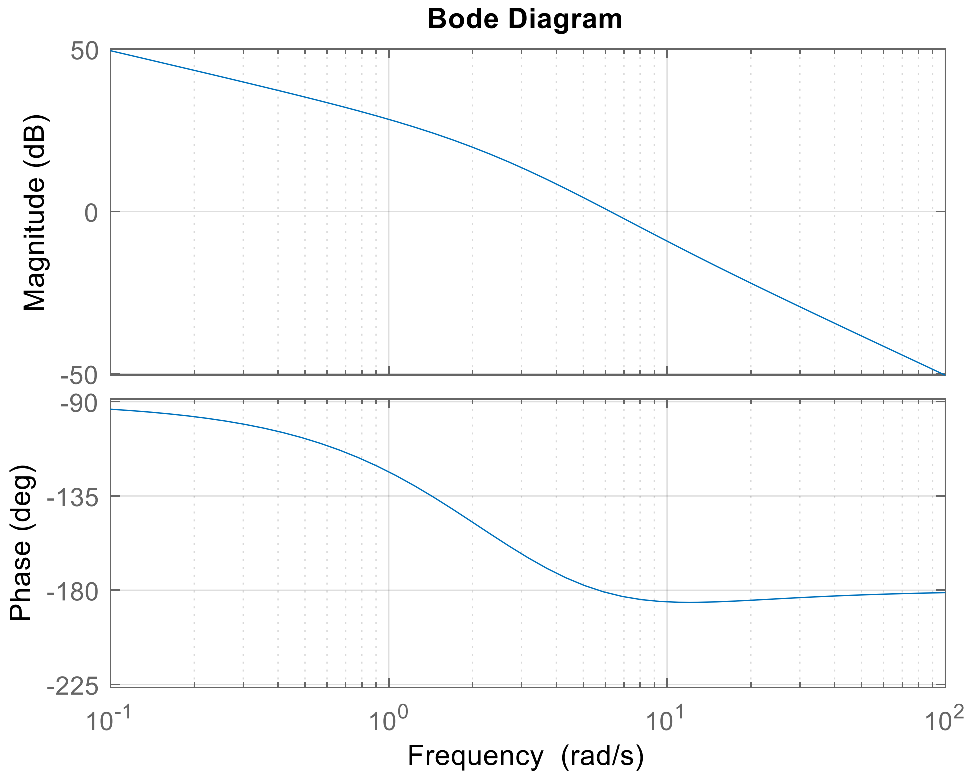
a) $G(s)=\dfrac{s+10}{s^2+6s+10}$

$$G(s)=\frac{10\left(\frac{s}{10}+1\right)}{10\left(\frac{s^2}{10}+\frac{6s}{10}+1\right)}=\frac{\left(\frac{j\omega}{10}+1\right)}{\left(\frac{(j\omega)^2}{10}+\frac{6j\omega}{10}+1\right)}$$



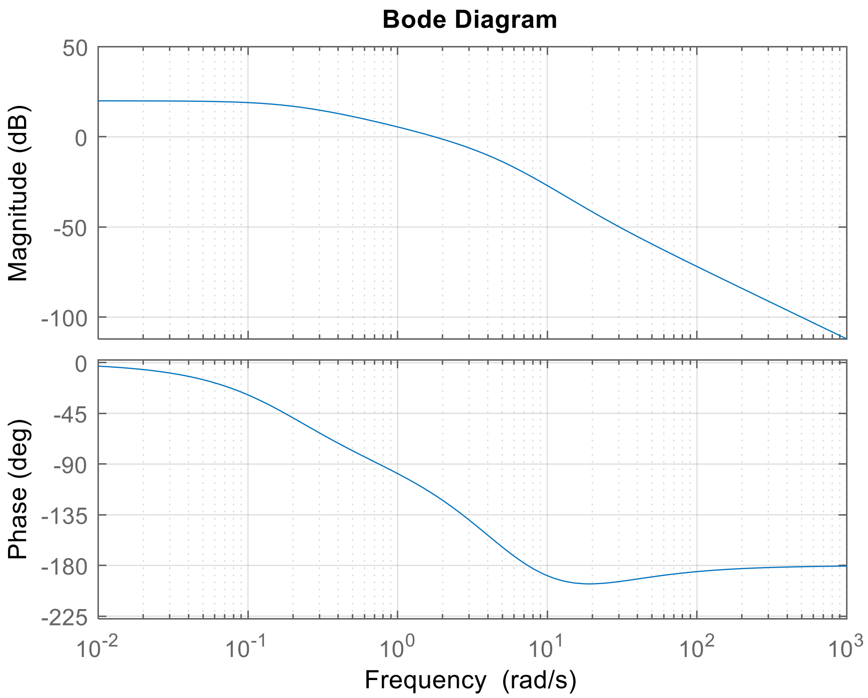
b) $G(s)=\dfrac{30(s+8)}{s(s+2)(s+4)}$

$$G(j\omega)=\frac{30\cdot 8\cdot\left(\frac{j\omega}{8}+1\right)}{4\cdot 2\cdot j\omega\cdot\left(\frac{j\omega}{2}+1\right)\cdot\left(\frac{j\omega}{4}+1\right)}$$

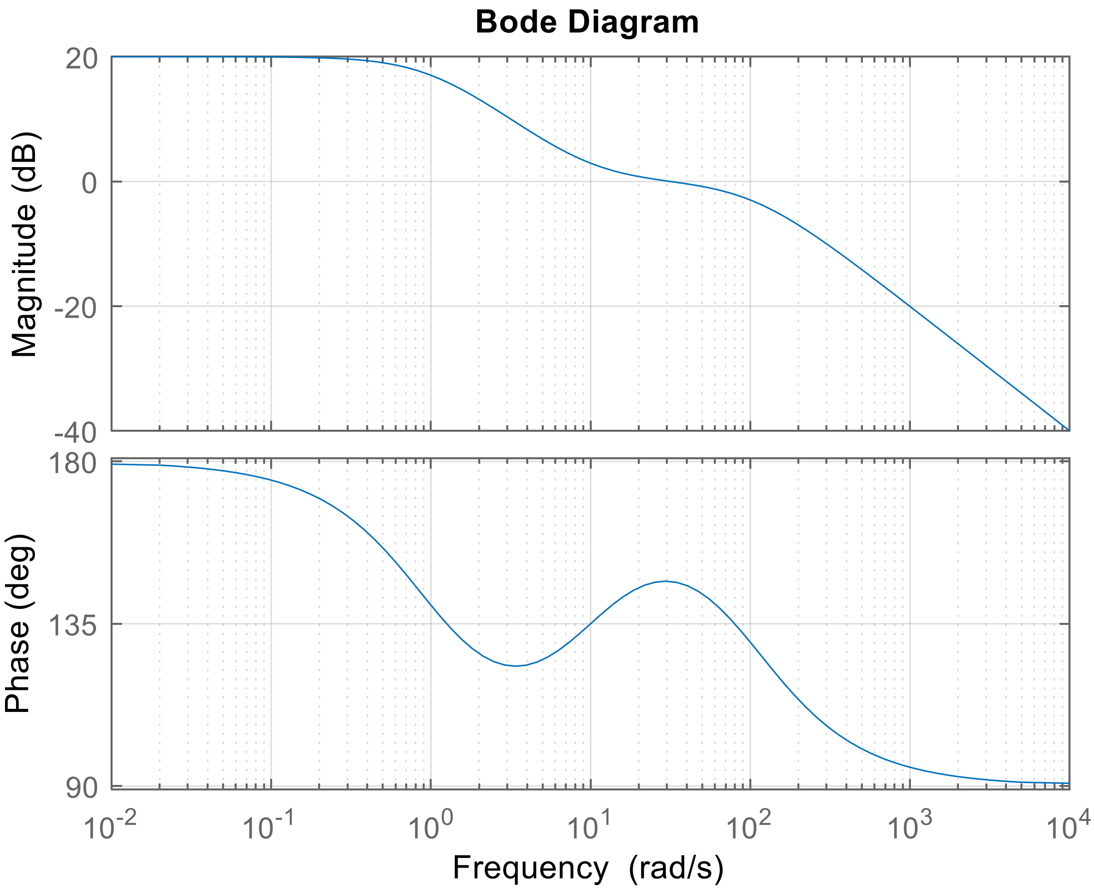


c) $G(s)=\dfrac{10(1+0.05\,s)}{(1+5\,s)(1+0.2\,s)^2}$

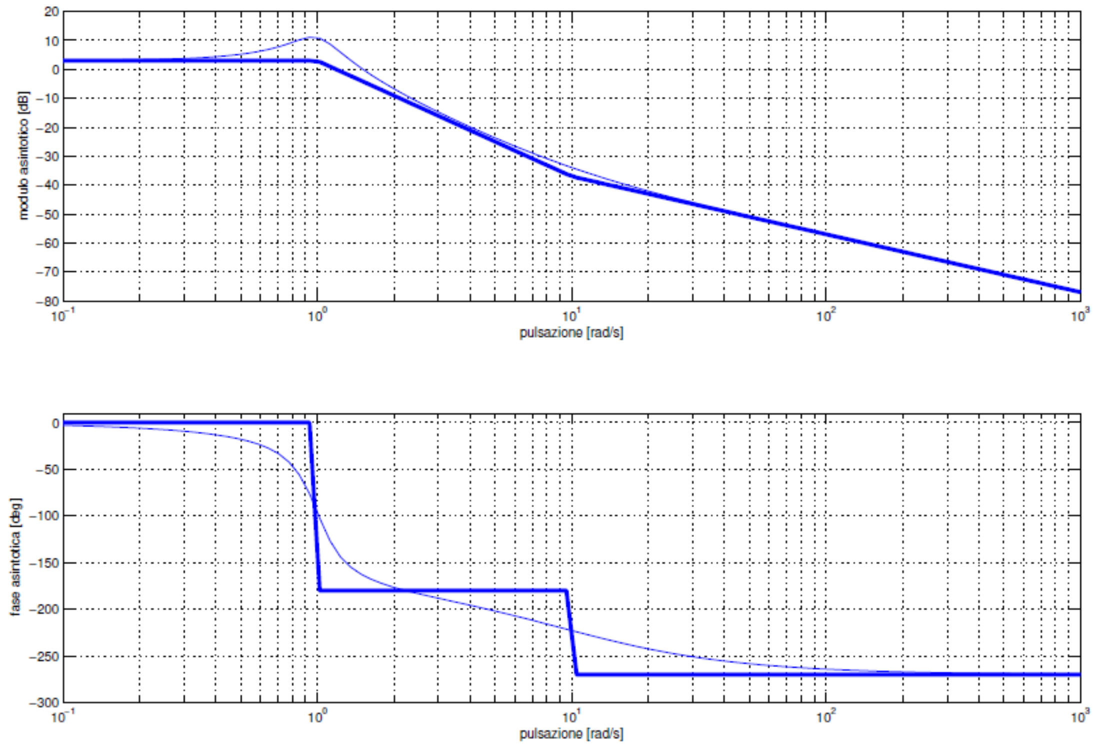
$$G(j\omega)=\frac{10(1+0.05\,j\omega)}{(1+5\,j\omega)(1+0.2\,j\omega)^2}=\frac{10\cdot\left(1+\dfrac{j\omega}{20}\right)}{\left(1+\dfrac{j\omega}{0.2}\right)\cdot\left(1+\dfrac{j\omega}{5}\right)^2}$$



Identify the transfer function corresponding to the Bode plots shown below:



$$G(s) = \frac{-10 \cdot \left(1 + \frac{s}{10}\right)}{(1+s)\left(1 + \frac{s}{100}\right)}$$



$$G(s) = \frac{k \cdot \left(1 - \frac{s}{10}\right)}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\xi}{\omega_n}s + 1}$$

The zero is on the RHP which can be seen from the negative phase that is introduced at ω=10.

Again, from bode plot, $\omega_n = 1$ and ξ can be estimated by,

$$\xi \approx \frac{\varphi_m}{100} = \frac{20}{100} = 0.2$$

or

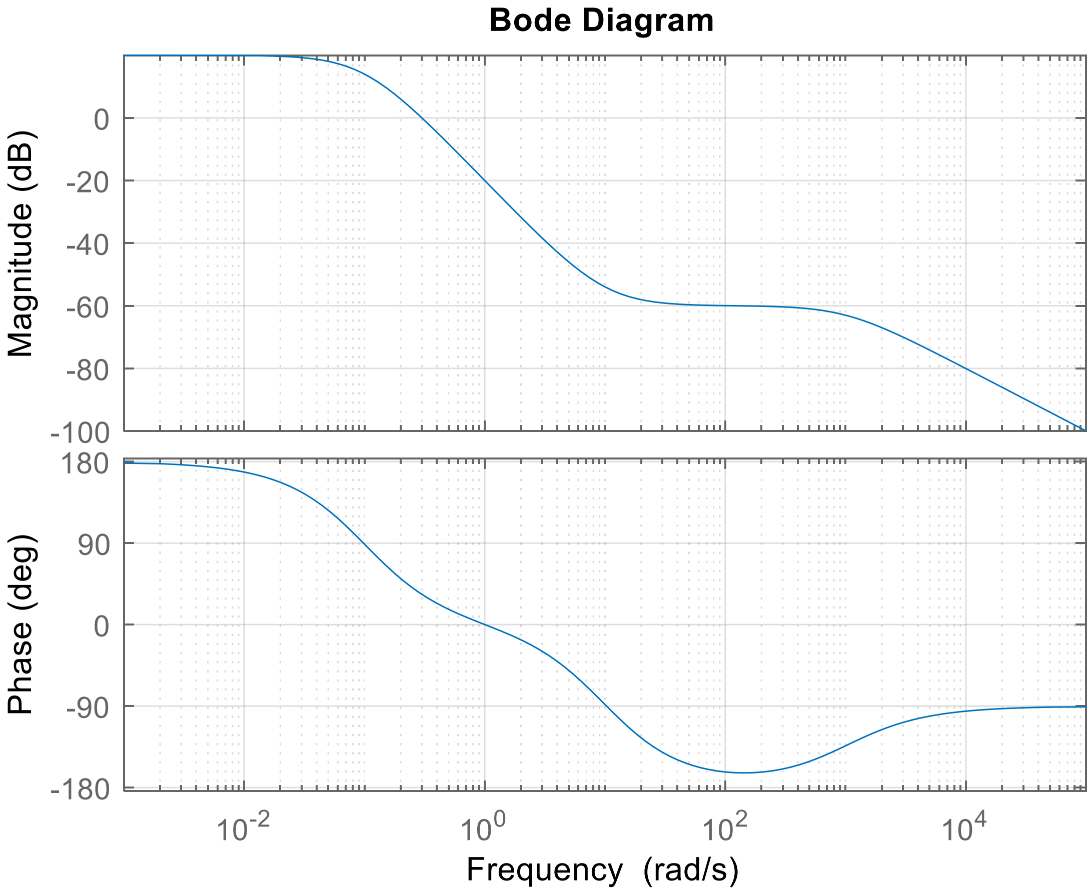
$$|L(j\omega_n)| = \frac{1}{2\xi}$$

Note that the bode magnitude is in dB, therefore it should be transformed to be able to by used by the above formula,

Finally, the constant k can be estimated by the gain of the bode plot as:

$$20 \log k \approx 3$$

Hence, k=1.41.



$$G(s)=\frac{-10\cdot\left(1-\frac{s}{10}\right)^2}{\left(1+\frac{s}{0.1}\right)^2\cdot\left(1-\frac{s}{1000}\right)}$$

Determine the polar plot of the following transfer function:

$$G(s) = \frac{30}{s^2 + 4s + 3}$$

$$G(s) = \frac{30}{s^2 + 4s + 3} = \frac{30}{(s+3)(s+1)} = \frac{30/3}{\left(\frac{s}{3}+1\right)(s+1)}$$

Frequency response: $G(j\omega) = \dfrac{10}{\left(\frac{j\omega}{3}+1\right)(j\omega+1)}$

Then we need to find the magnitude and angle of the frequency response,

Complex numbers magnitude and angle:

$$a + jb$$

$$|M| = \sqrt{a^2 + b^2}$$

$$\angle = \tan^{-1}\left(\frac{b}{a}\right)$$

Therefore,

$$|G(j\omega)| = \frac{10}{\left(\sqrt{\left(\frac{\omega}{3}\right)^2 + 1}\right)\left(\sqrt{\omega^2 + 1}\right)}$$

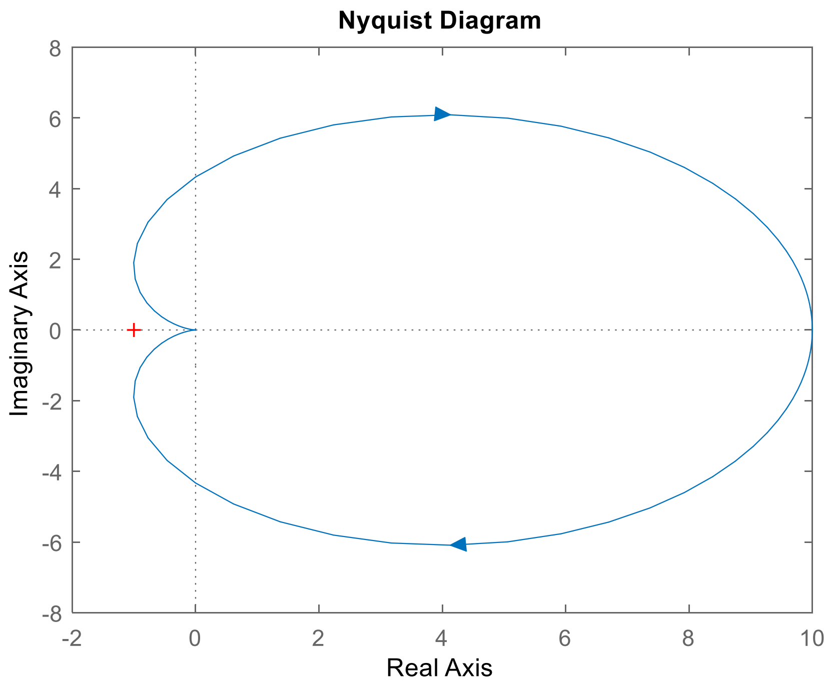
$$\angle G(j\omega) = \tan^{-1}\left(\frac{0}{10}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) - \tan^{-1}(\omega)$$

Then, we should apply ω=0 and ω=∞ and determine the magnitude and angle.

For ω=0→ $|G(j\omega)| = 10 \quad \angle G(j\omega) = 0$

For ω=∞→ $|G(j\omega)| = 0 \quad \angle G(j\omega) = 0 - 90 - 90 = -180$

To find the points between zero and infinity we should substitute various values for ω (e.g ω=0.1,0.2,0.5)



Nyquist Diagram
Imaginary Axis
Real Axis
8
6
4
2
0
-2
-4
-6
-8
-2
0
2
4
6
8
10