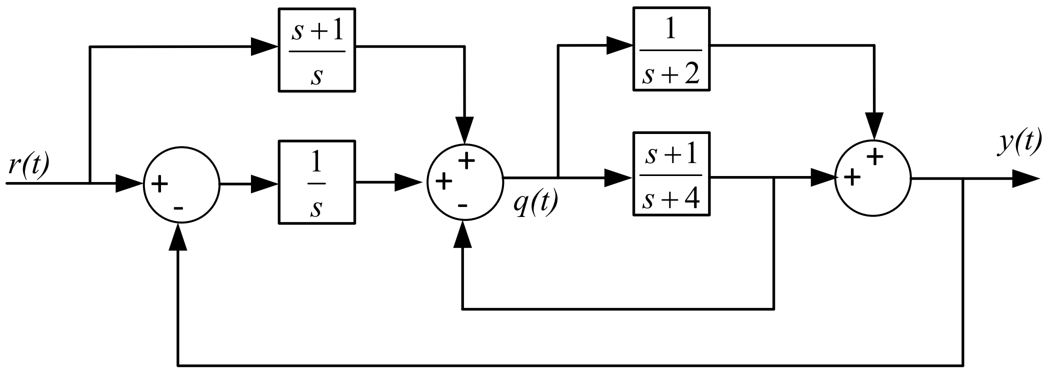
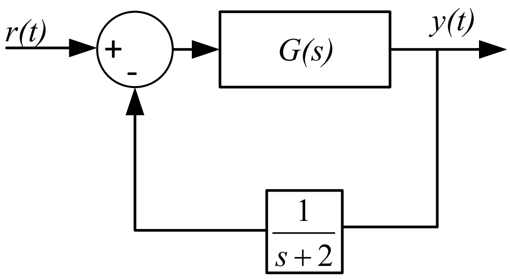
Tutorial 5

ECE 804-Block Diagrams

1. Determine G(s) in the simplest form in a such way that both systems shown below to have the same transfer function from r(t) to y(t).





From the second diagram:

$$Y = G\left(R - \frac{1}{s+2}Y\right)$$

From the first diagram:

$$Y = \frac{1}{s+2}Q + \frac{s+1}{s+4}Q = \frac{s^2+4s+6}{s^2+6s+8}Q \quad (1)$$

$$Q = \frac{s+1}{s}R + \frac{1}{s}(R-Y) - \frac{s+1}{s+4}Q$$

$$\frac{2s+5}{s+4}Q = \frac{1}{s}\left((s+2)R - Y\right)$$

$$\Rightarrow Q = \frac{(s+4)(s+2)}{s(2s+5)}\left(R - \frac{1}{s+2}Y\right) \quad (2)$$

(2) $\rightarrow$ (1)

$$Y = \frac{s^2+4s+6}{s^2+6s+8}\frac{(s+4)(s+2)}{s(2s+5)}\left(R - \frac{1}{s+2}Y\right)$$

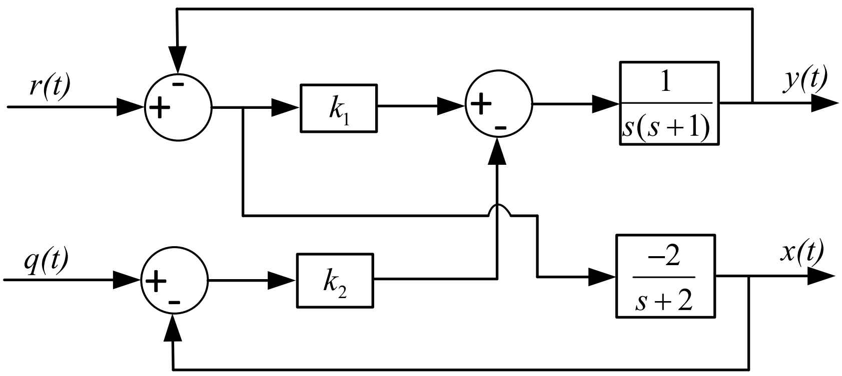
$$Y = \frac{s^2+4s+6}{s(2s+5)}\left(R - \frac{1}{s+2}Y\right)$$

$$Y = \frac{s^2+4s+6}{s(2s+5)}\frac{Y}{G} \Rightarrow G = \frac{s^2+4s+6}{s(2s+5)}$$

2. For the feedback system shown below:

a. Determine the transfer function from r(t) to y(t)

b. Determine the transfer function from q(t) to x(t)



$$Y = \frac{1}{s(s+1)}\left(k_1(R-Y) - k_2(Q-X)\right)$$

$$X = -\frac{2}{s+2}(R-Y)$$

a. q(t) should be set to 0

$$Y = \frac{1}{s(s+1)}\left(k_1(R-Y) - k_2\frac{2}{s+2}(R-Y)\right)$$

$$\Rightarrow Y = \frac{1}{s(s+1)(s+2)}\left(k_1(s+2) - 2k_2\right)(R-Y)$$

$$\Rightarrow (s^3+3s^2+2s+k_1(s+2)-2k_2)Y = (k_1(s+2)-2k_2)R$$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{k_1 s + 2k_1 - 2k_2}{s^3+3s^2+(2+k_1)s+2k_1-2k_2}$$

b. r(t) should be set to 0

$$s(s+1)Y = -k_1Y - k_2Q + k_2X \ (1)$$

$$X = \frac{2}{s+2}Y \ (2)$$

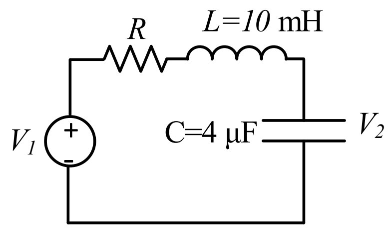
$$(1) \rightarrow (s^2 + s + k_1)Y = -k_2Q + k_2X$$

$$(2) \rightarrow Y = \frac{s+2}{2}X$$

$$(2) \rightarrow (1) \Rightarrow (s^2 + s + k_1)\frac{s+2}{2}X = -k_2Q + k_2X$$

$$\Rightarrow \frac{X(s)}{Q(s)} = \frac{-2k_2}{(s^2 + s + k_1)(s+2) - 2k_2}$$

3. Consider the circuit shown below:



a. Determine the transfer function of the system in terms of R (G(s)=V_2(s)/V_1(s)).

$$V_2(s) = \frac{V_1(s)\cdot \frac{1}{Cs}}{\frac{1}{Cs} + R + Ls} \Rightarrow \frac{V_2(s)}{V_1(s)} = G(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{25\cdot 10^6}{s^2 + 100Rs + 25\cdot 10^6}$$

b. Determine the natural frequency and damping factor of the system in terms of R.

To determine the natural frequency and damping factor, the denominator should be in the following form

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 100Rs + 25\cdot 10^6$$

Therefore,

$$\omega_n^2 = 25\cdot 10^6 \Rightarrow \omega_n = 5000\, rad/s$$

$$2\zeta\omega_n = 100R \Rightarrow \zeta = \frac{R}{100}$$

c. If the maximum overshoot on the system should be less than 25% what value of R should be selected?

The formula the is used to calculate the maximum overshoot is:

$$\Delta = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \le 0.25$$

Taking ln() in both sides,

$$-\frac{\zeta\pi}{\sqrt{1-\zeta^2}} \le \ln(0.25)$$

Then, square both terms

$$\frac{\zeta^2\pi^2}{1-\zeta^2} \le \ln^2(0.25)$$

$$\Rightarrow \frac{1}{\zeta^2} - 1 \le \frac{\pi^2}{\ln^2(0.25)}$$

$$\Rightarrow \frac{1}{\zeta^2} \le \frac{\pi^2}{\ln^2(0.25)} + 1$$

$$\Rightarrow \zeta^2 \ge \frac{1}{\frac{\pi^2}{\ln^2(0.25)} + 1} \ge 0.1629$$

$$\Rightarrow \zeta \ge 0.404 \Rightarrow \frac{R}{100} \ge 0.404 \Rightarrow R \ge 40.4\Omega$$

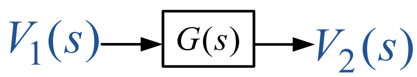
d. Determine the final value of the step response.

To determine the final value when the input is a step signal, we can use the final value theorem. The R is assumed to be the lower value that found in the previous question.

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{25\cdot 10^6}{s^2 + 4040s + 25\cdot 10^6} \cdot \frac{1}{s} s = 1$$

e. Determine the response of the system to an input $V_1(t)=10\sin(2500t)$.

If G(s) is the transfer function of the system, then the schematic diagram to an input V_1 and output V_2 is:



In case the input is sinusoidal, the output will also be sinusoidal but with different amplitude and phase difference. In general, the form will be:

$$V_2(s) = 10\left|G(j\omega)\right| \sin(2500t + \angle G(j\omega))$$

Therefore, we need to calculate the magnitude and phase angle of the transfer function.

$$G(2500j) = \frac{25\cdot 10^6}{(2500j)^2 + (4040\cdot(2500j)) + 25\cdot 10^6}$$

$$= \frac{25\cdot 10^6}{-6.25\cdot 10^6 + 10.1\cdot 10^6 j + 25\cdot 10^6} = 1.03 - 0.55j$$

$$\Rightarrow \left|G(2500j\right| = \sqrt{1.03^2 + (-0.55)^2} = 1.17$$

$$\angle G(2500j) = \tan^{-1}\left(\frac{-0.55}{1.03}\right) = -28.31^\circ = -0.494\,rad$$

Therefore the response of the system to the input will be:

$$V_2(s) = 10\cdot(1.17)\sin(2500t - 0.494)$$