Tutorial 3

ECE 804- *State Space Models*

1. Determine the stability properties of the following systems:

a. $$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 3x_2(t) \\ \dot{x}_2(t) = -5x_1(t) - 2x_2(t) + u(t) \end{cases}$$

b. $$\begin{cases} \dot{x}_1(t) = 6x_1(t) + 7x_2(t) + \mathrm{u(t)} \\ \dot{x}_2(t) = -2x_1(t) - x_2(t) \end{cases}$$

c. $$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 7x_2(t) \\ \dot{x}_2(t) = 5x_2(t) + u(t) \end{cases}$$

d. $$\begin{cases} \dot{x}_1(t) = 2x_1(t) + 7x_2(t) \\ \dot{x}_2(t) = 4x_1(t) + 5x_2(t) + u(t) \\ \dot{x}_3(t) = -5x_3(t) \end{cases}$$

2. Consider the linear system described by the state equations:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2x_1(t) - 3x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases}$$

a. Write the state equations in matrix form $\dot{\mathbf{x}} = A\mathbf{x} + Bu; y = C\mathbf{x} + Du$ for suitable matrices *A, B, C, D* and determine the equilibrium states associated with constant input *u(t)=2,* $\forall t \geq 0$

b. Analyze the stability properties of all the equilibrium states.

c. Determine the state transition matrix e^{At} where matrix A is given in your answer to Q2-a.

d. Determine x(t) and y(t) when u(t) is the step function and $x(0)=[1\ 0]^T$.

e. Repeat c with u(t)=sin2t.