

# Optimization of CIS

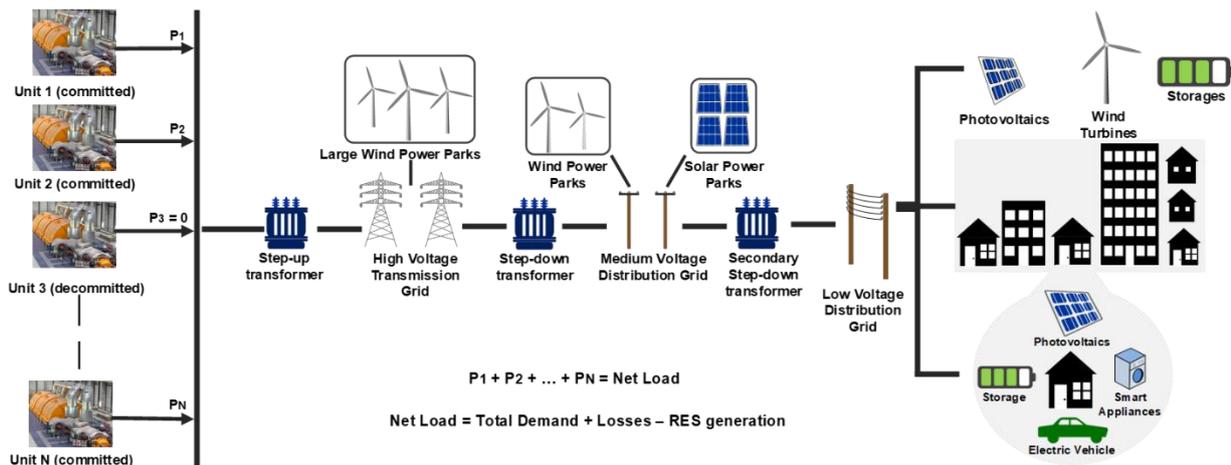
## ECE 802

### Tutorial 1 – Economic dispatch via linear programming

L. Tziovani and A. Astolfi

#### Economic Dispatch

Consider a power system with  $N$  generating units as shown in the figure below. In this system there are multiple types of fuel inputs used to operate the power plant (e.g., gas, heavy fuel oil, diesel.). Therefore, the generating units have different fuel consumption and characteristics. The aim of the economic dispatch is to determine the power output among the committed generating units to serve the system net load at minimum total cost.



**Definition:** The economic dispatch is an optimization problem which defines the power output of the committed generating units to minimize the total operational cost of the system, subjects to operational and security constraints.

**Linear and Quadratic Cost Functions:** The cost function of each generating unit is represented by linear, quadratic or cubic functions, with the quadratic function the most commonly used description. The cost function determines the operational cost of each unit as a function of the power produced by the unit.

Linear Cost Function:

$$C(P) = a + bP \quad \text{€/h}$$

Quadratic Cost Function:

$$C(P) = a + bP + cP^2 \quad \text{€/h}$$

Where  $a$  ( $\frac{\text{€}}{\text{h}}$ ), ( $\frac{\text{€}}{\text{MWh}}$ ), and  $c$  ( $\frac{\text{€}}{(\text{MWh})^2}$ ), are the cost coefficients, and  $P$  (MW) is the power produced by the generating unit.

**Maximum and Minimum Power:** The produced power of a committed unit must be within a minimum ( $\underline{P}$ ) and a maximum ( $\overline{P}$ ) value, based on its technical characteristics, that is

$$\underline{P} \leq P \leq \overline{P}$$

**Economic Dispatch vs Unit Commitment:** The unit commitment optimization problem defines which units of the power system must be committed (ON) and how much power it has to produce in order to minimize the total operating cost of the system over a time horizon (usually a day ahead). Therefore, the economic dispatch is a sub-problem of the unit commitment. The unit commitment is a mixed-integer programming problem which is very difficult to solve for large scale systems due to the fact that the computation time increases exponentially as the number of decision variables increases.

## Linear Programming

A linear programming model demands that the objective function and the constraints are described by linear expressions such as,  $3x_1 + 5x_2$ . Terms such as  $x^3$ ,  $e^x$ ,  $x_1x_2$  are non-linear. Linear programming models can be solved efficiently even for large scale problems with thousands variables. This is the reason why non-linear models are often converted or approximated by linear models. Note that linear programming problems are convex problems.

**Example 1:** Solve the linear programming model formulated below. Note that a two variable-problem can be studied and solved geometrically.

Maximize  $F$ :

$$F = 3x_1 + 2x_2$$

subject to

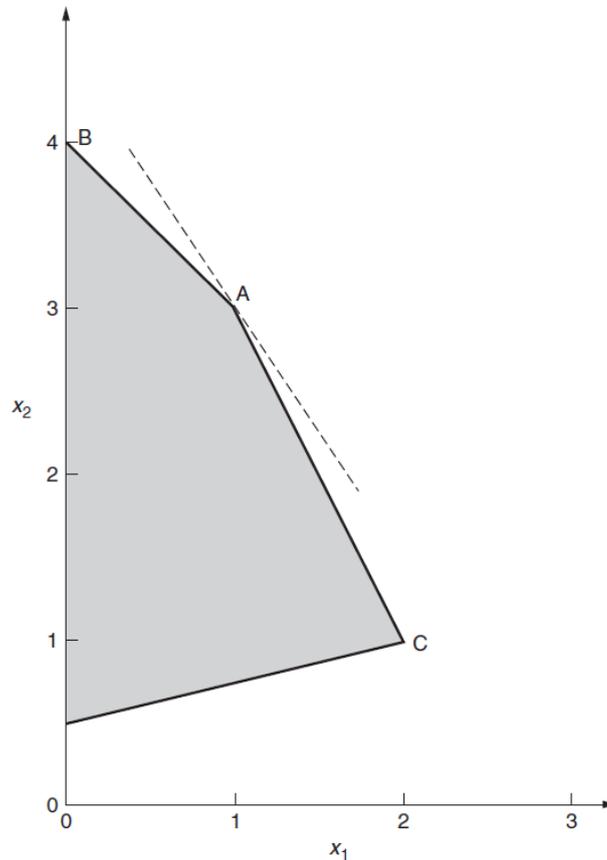
$$x_1 + x_2 \leq 4$$

$$2x_1 + x_2 \leq 5$$

$$-x_1 + 4x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

**Solution:** It is obvious that the optimal solution always lies on the boundary of the feasible (shaded) region. In this case, the point A is the optimal solution giving  $F = 9$ ,  $x_1 = 1$  and  $x_2 = 3$ .



Example 2: Solve the linear programming problem of Example 1 in Matlab using the linprog optimization toolbox of Matlab (<https://www.mathworks.com/help/optim/ug/linprog.html>).

## linprog

Solve linear programming problems

Linear programming solver

Finds the minimum of a problem specified by

$$\min_x f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

$f$ ,  $x$ ,  $b$ ,  $beq$ ,  $lb$ , and  $ub$  are vectors, and  $A$  and  $Aeq$  are matrices.

## Syntax

```
x = linprog(f,A,b)
x = linprog(f,A,b,Aeq,beq)
x = linprog(f,A,b,Aeq,beq,lb,ub)
x = linprog(f,A,b,Aeq,beq,lb,ub,options)
x = linprog(problem)
[x,fval] = linprog(__)
[x,fval,exitflag,output] = linprog(__)
[x,fval,exitflag,output,lambda] = linprog(__)
```

Solution:

Step 1: Reformulate the problem as a minimization problem and change the inequality constraints from  $\geq$  to  $\leq$ , to obtain the following equivalent formulation.

Minimize

$$F_n = -3x_1 - 2x_2$$

subject to

$$x_1 + x_2 \leq 4$$

$$2x_1 + x_2 \leq 5$$

$$x_1 - 4x_2 \leq -2$$

$$x_1, x_2 \geq 0$$

Step 2: Code the problem in Matlab and call the solver.

```
% Linear programming model
%Min F=-3x1 - 2x2
%s.t
%x1+x2<=4
%2x1+x2<=5
%x1-4x2<=-2
%x1,x2>=0

%variables order: x1, x2

%objective function
fn=[-3 -2];

% bounds of the variables
lb=[0 0]; %lower
ub=[inf inf]; %upper

%Inequality constraints
A=[ 1  1;
    2  1;
    1 -4];
b=[ 4 ;
    5;
    -2];

%Equality constraints (There are no equality constraints)
Aeq=[];
beq=[];

%call the solver
x=linprog(fn, A,b,Aeq,beq,lb,ub);

Fn=-3*x(1)-2*x(2); %Value of objective
F=-Fn; %%Value of objective for the maximization problem
```

Results:  $x=[1 \ 3]$ ,  $F_n=-9$ ,  $F=9$

**Example 3 – Economic Dispatch:** Consider a power system with the following eight committed units:

1 GT unit:  $C(P) = 710 + 60P$  €/h,  $5 \leq P \leq 30$  MW

3 ST units:  $C(P) = 670 + 40P$  €/h,  $25 \leq P \leq 70$  MW

2 ICE units:  $C(P) = 150 + 38P$  €/h,  $7 \leq P \leq 28$  MW

2 ST units:  $C(P) = 870 + 37P$  €/h,  $50 \leq P \leq 140$  MW

Find the produced power (P) of each generating unit to minimize the total operational cost of the system, while satisfying the power balance and the generation constraints. Solve the problem for load demand= 450 MW, and for load demand= 580 MW. **Note:** The total generation must be equal to the load demand (power balance constraint).

**Solution:** Problem Formulation

**Note:** The constant coefficients do not affect the solution of the problem. So, they can be removed and can be added to the total cost after the solution of the optimization problem.

The objective function,  $C_T$ , is the summation of the cost function of the units (N=8) and the objective is to minimize the total cost. The problem is therefore

$$\min C_T = 60P_1 + 40P_2 + 40P_3 \dots + 38P_8 = \sum_{i=1..N} C_i(P_i)$$

subject to

$$\text{Power Balance Constraint: } P_1 + P_2 \dots + P_8 = \sum_{i=1..N} P_i = \text{Load Demand}$$

$$\text{Generation Stable Limits: } P_{i,min} \leq P_i \leq P_{i,max}, \forall i = 1 \dots N$$

```
% Economic Dispatch using Linear programming model
clear all;
clc;
%variables order: P1, P2, ... P8
%objective function
F=[60 40 40 40 38 38 37 37];

% bounds of the variables
lb=[5 25 25 25 7 7 50 50]; %lower
ub=[30 70 70 70 28 28 140 140 ]; %upper

%Inequality constraints (There are no inequality constraints)
A=[];
b=[];

%Equality constraints
Aeq=ones(1,8); %[1 1 1 ...]
beq=450; %load demand

%call the solver
x=linprog(F, A,b,Aeq,beq,lb,ub);

F_linear_terms=60*x(1)+40*x(2)+40*x(3)+40*x(4)+38*x(5)+38*x(6)+37*x(7)+37*x(8);
F_constant_terms=710+670*3+150*2+870*2;

Total_operational_cost = F_linear_terms + F_constant_terms;
```

For load demand=450:

$X=[5\ 25\ 25\ 59\ 28\ 28\ 140\ 140]$ , Total Operational cost= 21908 €

For load demand=580:

**No feasible solution**

Practice exercise:

An engineering factory can produce five types of product (PROD 1, PROD 2, . . . , PROD 5) by using two production processes: grinding and drilling. After deducting raw material costs, each unit of each product yields the following contributions to profit:

PROD 1	PROD 2	PROD 3	PROD 4	PROD 5
£550	£600	£350	£400	£200

Each unit requires a certain time on each process. These are given below (in hours). A dash indicates when a process is not needed.

	PROD 1	PROD 2	PROD 3	PROD 4	PROD 5
Grinding	12	20	–	25	15
Drilling	10	8	16	–	–

In addition, the final assembly of each unit of each product uses 20 hours of an employee's time. The factory has three grinding machines and two drilling machines and works a six-day week with two shifts of 8 hours on each day. Eight workers are employed in assembly, each working one shift a day. The problem is to find how much of each product is to be manufactured so as to maximize the total profit contribution. Formulate the problem as a linear programming problem and use Matlab to solve it.