ECE 802 - Optimization of CIS

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COURSE-WORK

The final report of the Part I should be submitted by the 27th of September while the report of Part II should be submitted by the 21th of October 2019. Both reports should be submitted electronically to ltziov01@ucy.ac.cy

Part I - Unconstrained Optimization [60 marks]

To compare the performance of optimisation methods it is customary to construct *test* functions and then compare the behavior of various optimization algorithms for such functions.
One, often used, test function is the so-called Rosenbrock function, *i.e.*

$$v(x, y) = 100(y - x^2)^2 + (1 - x)^2.$$

A1) Compute analytically all stationary points of the function $v(x, y)$ and verify if they are minimizers/maximizers/saddle points. [3 marks]

A2) Plot (using Matlab or a similar SW) the level sets of the function $v(x, y)$. [3 marks]

A3) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the gradient method (see Section 2.5) with Armijo line search (see Section 2.4.2). [7 marks]

A4) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using Newton method (see Section 2.6) with and without Armijo line search. [7 marks]

A5) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Polak-Ribiere algorithm (see Section 2.7.3) with Armijo line search. [9 marks]

A6) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Broyden-Fletcher-Goldfarb-Shanno algorithm (see Section 2.8) with Armijo line search. [9 marks]

A7) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the simplex method (see Section 2.9). Consider modifying the method in Section 2.9 to improve convergence properties of the algorithm. [10 marks]

A8) *Run* the minimization procedures written in points A3) to A7) with initial point $(x_0, y_0) = (-3/4, 1)$.

A8a) Plot, on the (x, y)-plane, the sequences of points generated by each algorithm. Are these sequences converging to a stationary point of $v(x, y)$? [3 marks]

A8b) For a sequence $\{x_k, y_k\}$ consider the *cost*

$$J_k = \log\left((x_k - 1)^2 + (y_k - 1)^2\right).$$

Plot, for each of the sequences generated by the above algorithms, the cost J_k as a function of k. Use such a plot to assess the speed of convergence of each of the considered algorithms. [9 marks]

Part II - Constrained Optimization [40 marks]

B1) A food is manufactured by refining raw oils and blending them together. The raw oils come in

Vegetable oils	VEG 1
Vegetable oils	VEG 2
Non-vegetable oils	OIL 1
Non-vegetable oils	OIL 2
Non-vegetable oils	OIL 3

Vegetable oils and non-vegetable oils require different production lines for refining. In any month it is not possible to refine more than 200 tons of vegetable oil and more than 250 tons of non-vegetable oils. There is no loss of weight in the refining process and the cost of refining may be ignored.

There is a technological restriction of hardness in the final product. In the units in which hardness is measured, this must lie between 3 and 6. It is assumed that hardness blends linearly. The costs (per ton) and hardness of the raw oils

	VEG 1	VEG 2	OIL 1	OIL 2	OIL 3
Cost (€)	110	120	130	110	115
Hardness	8.8	6.1	2.0	4.2	5.0

The final product sells for 150 per ton. How should the food manufacturer make their product in order to maximize their net profit? Formulate the problem as a linear programming and use solver linprog to solve it.

[10 marks]

B2) M computers are available to execute N jobs. The job i requires T_{ij} units of computation time in order to be executed by the computer j, while the energy consumption is E_{ij} (the matrices E and T are indicated below). Each job must be executed by one computer, while each computer executes the jobs in series. All the jobs must be executed and the aim is to minimize the total energy consumption. Also, the total computation time of each computer must be higher than t units, but each computer must not execute more than K jobs. Formulate the problem as an integer programming and solve it for $M = 5, N = 12, K = 4, t = 60$ using the Gurobi solver. Which is the optimal job assignment and what is the minimum energy consumption?

[15 marks]

$$\mathbf{E} = \begin{bmatrix} 1.2 & 4.3 & 3.8 & 3.9 & 1.4 & 4.9 & 2.3 & 4.5 & 3.3 & 4.4 & 1.2 & 3.4 \\ 1.6 & 3.5 & 4 & 3.1 & 2.6 & 2.4 & 1.7 & 2.8 & 3 & 4.3 & 1.4 & 4.3 \\ 1 & 1.4 & 4.2 & 2.3 & 3.2 & 1.9 & 2.1 & 1.3 & 3.2 & 1.7 & 2.2 & 3.5 \\ 2 & 1 & 4.4 & 2 & 4.9 & 2.1 & 4.3 & 4.6 & 3.7 & 1.9 & 4.1 & 2.5 \\ 4.6 & 3.2 & 3.1 & 4.8 & 4.8 & 4.5 & 2.5 & 3 & 4.4 & 3.9 & 4.7 & 4 \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} 36 & 46 & 34 & 25 & 29 & 17 & 15 & 29 & 38 & 50 & 39 & 50 \\ 37 & 39 & 25 & 13 & 11 & 11 & 32 & 50 & 23 & 44 & 15 & 25 \\ 42 & 13 & 41 & 31 & 16 & 44 & 49 & 25 & 31 & 14 & 45 & 44 \\ 36 & 50 & 49 & 30 & 15 & 22 & 31 & 45 & 40 & 41 & 22 & 34 \\ 13 & 26 & 14 & 34 & 31 & 22 & 47 & 37 & 42 & 47 & 42 & 24 \end{bmatrix}$$

B3) Consider a power system with eight committed units. The cost functions of the units are given below:

$$1 \text{ unit: } C(P) = 710 + 60P + 0.37P^2 \text{ €/h}, \quad 5 \leq P \leq 30 \text{ MW}, \quad R_{max} = 4 \text{ MW}$$

$$3 \text{ units: } C(P) = 670 + 40P + 0.4P^2 \text{ €/h}, \quad 25 \leq P \leq 70 \text{ MW}, \quad R_{max} = 5 \text{ MW}$$

$$2 \text{ units: } C(P) = 150 + 38P + 0.28P^2 \text{ €/h}, \quad 7 \leq P \leq 28 \text{ MW}, \quad R_{max} = 5 \text{ MW}$$

$$2 \text{ units: } C(P) = 870 + 37P + 0.18P^2 \text{ €/h}, \quad 50 \leq P \leq 140 \text{ MW}, \quad R_{max} = 10 \text{ MW}$$

Each unit has a maximum operating reserve (R_{max}) and the available operating reserve of each unit is defined as the minimum between the R_{max} and the difference between the maximum power and the generating power (P) of the unit. For example, when the first unit operates at 20 MW then the available operating reserve of this unit is 4 MW, but when the unit operates at 28 MW then the available operating reserve of this unit is 2 MW. The total available operating reserve from all the committed units must be higher than the total system reserve (SR)

Find the produced power (P) of each generating unit, in order to minimize the total operational cost of the system, while satisfying the power balance (generation = demand), reserve and the generation constraints. Formulate the problem as a quadratic and use solver quadprog to solve the problem for load demand = 418 MW and $SR = 42$ MW.

[15 marks]

Gurobi optimizer guide (pages 6-10 and 80-83)
`https://www.gurobi.com/wp-content/plugins/hd_documentations/content/pdf/quickstart_windows_8.1.pdf`