Optimization of CIS

ECE 802

Tutorial 3 – Energy management in Buildings:
Optimization of a PV-Storage System

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Introduction

The penetration of renewable energy sources into the power system is expected to increase rapidly in the next years due to the European targets which aim to increase the share of renewable energy in the total energy consumption to 32% by 2030. However, the high penetration of renewable sources may create challenges in the operation and stability of the power system due to the intermittent nature of these sources. Technologies, such as energy storage systems can solve these issues by maintaining the balance of supply and demand, and supporting the stability and the frequency of the grid. To date, battery storage is the most accepted technology among the alternative energy storage systems which are used in a wide range of applications, such as electric vehicles and storage of electric energy produced by renewable energy sources. Moreover, batteries can reduce the cost of electricity in commercial and residential buildings by absorbing and storing energy from the grid during off-peak periods and injecting it during high-demand periods when prices are very high. The aim of this tutorial is to design an algorithm for the energy management of a building. The algorithm must decide when to buy or sell power to the power grid, and when to charge or discharge the storage in order to minimize the cost of electricity for the building for the next day using predicted input data.

System Description

The system under study is presented in the figure below. The system is essentially a non-residential building that is interconnected to the power grid. The non-residential building is composed of three photovoltaic-storage-inverter sub systems connected in parallel and the building loads. Each sub-system consists of 20 PV panels connected in series with a maximum power of 5 kW, a flywheel storage with 6 kWh usable capacity range and 3 kW maximum charging-discharging power. Further, a hybrid 8 kW inverter is included in the sub-system. The total aggregated PV capacity of the system is 15 kW, the total usable capacity of the flywheels storage is 18 kWh with 9 kW maximum charging-discharging rate, and the total maximum power of the inverters is 24 kW. The total maximum power from the PV and the flywheel storage can reach 24 kW which is equal to the maximum power of the inverters. As a result, there is no power limitation on the total power from the PV and the storage due to the inverter maximum power.
The bi-directional power flow between the AC and DC side is allowed through the hybrid inverter. Power from the grid can be absorbed to feed the load or even to charge the flywheel storage on the DC side. The flywheel storage can also be charged directly from the power generated by the photovoltaics through the DC-link. The PV output power and the discharging power of the storage is consumed by the load and the extra power is injected into the power grid. Assume that the system is ideal, and power losses are ignored (In real systems, energy storage systems and inverters present energy losses).

The forecasted input data for every 15-minute of the next day are presented in the figure below. More specifically, the produced power of the PV, the load demand of the building and day-ahead electricity pricing are presented. The day-ahead electricity pricing is assumed to be the same for both electricity purchase and selling.

Exercise: Formulate the energy management problem as a mathematical programming problem and then solve the resulting problem in Matlab using an optimization solver. The algorithm must decide when to buy or sell power to the power grid, and when to charge or discharge the storage in order to
minimize the cost of the electricity bill of the building for the next day using predicted input data. Assume that the storage is initially empty, and consider the energy limitation of the storage (the stored energy must be less or equal than the maximum capacity of the storage in each time interval). Plot the decisions (buying and selling power) for the power interaction with the power grid, and the storage operation over time (charging and discharging power). What is the daily cost of electricity? Compare the result with the daily cost of electricity in the case in which the storage does not exist.

**Solution**

**Notes:** Formulate the problem using the aggregated values of the PV and storage. Note that, since the inverters are ideal, they can be ignored.

**Problem Formulation**

The equivalent model for formulating the optimization problem is illustrated in the figure below. In this model, single components are used to represent the system components of each category by using their aggregated values. The arrows show the possible power flow into the system. Note that there is not a unique way to formulate a problem. Keep in mind that there are good and bad problem formulations based on the computation time that is needed to solve the problem, and the computation time is associated with the total number and with the type of the decision variables. For large scale systems a good problem formulation is always desirable.

**Variables:** To reduce the total number of variables the power absorbed and the power injected into the power grid is modeled with one continuous variable in each time interval $t$, where this variable can take positive values (power absorption) or negative values (power injection). Similarly, the charging and discharging power of the storage is modeled with one continuous variable in each time interval $t$, where this variable can take positive values (charging) or negative values (discharging).
There are two variables ($P_{\text{grid}}$, $P_{\text{stor}}$) in each time interval. For a time horizon of a day with 15 minute intervals, there are 96 time intervals. As a result, there are $2 \times 96 = 192$ variables.

**Coefficients:** $P_{\text{pv}}(t)$ and $L(t)$ are the forecasted input data (one value for each time interval). Also, $c(t)$ is the cost of buying or selling electricity in each time interval.

**Objective Function:**
The objective of this optimization problem is to minimize the cost of the electricity payment for the whole horizon of study $T$. Consequently, the cost of the imported power from the grid should be minimized and the profit from the power exported to the grid should be maximized. As such, the objective function can be expressed as follows,

$$
\min f = \sum_{t=1}^{T} \left( \frac{c(t)}{4} P_{\text{grid}}(t) \right)
$$

Note that the variable $P_{\text{grid}}(t)$ can take positive values when power is imported and negative values when power is exported. As a result, when the variables take positive values then a cost is added to the objective and when the variables take negative values then a profit (negative cost) is added to the objective. Also, note that the cost of buying or selling electricity ($c(t)$) is measured in €/kWh. Therefore, in order to be associated with 15-minutes time intervals, these values are divided by 4.

**Constraints:** The objective function is subject to the following constraints.

1) **Power Balance:** The input power in the Bus must be equal to the output power of the Bus. Note that $P_{\text{grid}}(t)$ is positive when power is absorbed, and $P_{\text{stor}}(t)$ is positive when the storage is charged. As a result the following constraints hold:

$$
P_{\text{pv}}(t) + P_{\text{grid}}(t) = P_{\text{stor}}(t) + L(t), \quad \forall t \in T
$$

$$
-\infty \leq P_{\text{grid}}(t) \leq \infty, \quad \forall t \in T
$$

$$
-9 \leq P_{\text{stor}}(t) \leq 9, \quad \forall t \in T
$$

2) **State of charge of the storage (energy limitation):** The state of charge of the storage ($SOC(t)$) is measured in kWh and it is expressed as the initial capacity of the storage ($IC$) minus the sum of the discharging power plus the sum of the charging power for all past and present time intervals (In this problem, $P_{\text{stor}}(j)$ is positive when the storage is charged, and negative when the storage is discharged). The charging and discharging power is divided by 4 due to the 15 min time slots. Note that $SOC(t)$ must be always between the minimum and maximum energy limits of the storage. In this problem, $IC=0$, $SOC = 0$ and $\overline{SOC} = 18$ kWh. As a result

$$
SOC(t) = IC + \sum_{j=1}^{t} \frac{1}{4} P_{\text{stor}}(j), \quad \forall t \in T
$$

$$
SOC(t) \geq SOC, \quad SOC(t) \leq \overline{SOC}, \quad \forall t \in T
$$

In order to avoid the usage of the variables $SOC(t)$, the above constraints can be written as:

$$
\underline{SOC} \leq IC + \sum_{j=1}^{t} \frac{1}{4} P_{\text{stor}}(j) \leq \overline{SOC}, \quad \forall t \in T
$$
OR

\[
\sum_{j=1}^{t} \frac{1}{4} P_{stor}(j) \leq SOC - IC, \quad \forall t \in T
\]

\[
-\sum_{j=1}^{t} \frac{1}{4} P_{stor}(j) \leq -SOC + IC, \quad \forall t \in T
\]

Matlab Code:

```matlab
% Linear Programming Model
clf; close all;

% Input data
IC=0; % Initial capacity of storage (MWh)
SOC_MIN=0; % Minimum state of charge of the storage (MWh)
SOC_MAX=10; % Maximum state of charge of the storage (MWh)
P_max=9; % Maximum discharging power of the storage (MW)
P_ch=9; % Maximum charging power of the storage (MW)

% *********************************************************************************************************************************************
path_data='C:\Users\lwiz01\Desktop\F8 optimization of CTR\Tutorial 3\inpuc_data';
LoadM=load(path_data,'AIarge'); % Load demand (MW) in each 15 minute of the next day
PV=load(path_data,'PVlarge'); % PV generation (MW) in each 15 minute of the next day
Cost=load(path_data,'Cost06'); % Cost of buying or selling power (Euro/kWh) in each 15 minute of the next day

time_intervals=6; % The total intervals of the horizon

% ******************************************************************************
% Variables: [Pgrid(1), Pgrid(2), ..., Pgrid(6), Petor(1), Petor(2), ..., Petor(6)] % 192 variables
% Objective Function
f=[c'/4, zeros(1, time_intervals)]; % Only the Pygrid variables are included in the objective function

% ******************************************************************************
% Bounds of the variables:
lb=[-inf*ones(1, time_intervals), -P_max*ones(1, time_intervals)]; % lower bounds
ub=[inf*ones(1, time_intervals), P_max*ones(1, time_intervals)]; % upper bounds

% ******************************************************************************
% Equality constraints:
% Pgrid(t)+Pstor(t)+E(t)-Pgrid(t) for each t (there are 96 equality constraints)
% Left hand side:
Aeq=[diag(ones(1, time_intervals))+1, -diag(ones(1, time_intervals))]; % The matrixes for the Pgrid and Petor variables are diagonal.
beq=[Load-PV]; % Vector right hand side of the equality

% ******************************************************************************
% Inequality constraints:
% 1) sum_Pstor(t) = SOC_MAX-IC for each t (there are 96 inequality constraints)
A=[zeros(1, time_intervals, time_intervals), 1/4*tril(ones(time_intervals))]; % The matrix for the Petor variables is a lower triangular.
b=[(SOC_MAX-IC)*ones(1, time_intervals, 1) - IC*ones(time_intervals, 1)]; % Vector right hand side of the inequality

% 2) sum_Pstor(t) = SOC_MIN+IC for each t (there are 96 inequality constraints)
A_tmp=[zeros(time_intervals, time_intervals), 1/4*tril(ones(time_intervals))]; % The matrix for the Petor variables is a lower triangular
b_tmp=[-(SOC_MIN+IC)*ones(time_intervals, 1) + IC*ones(time_intervals, 1)]; % Vector right hand side of the inequality

A=[A; A_tmp]; % One matrix for all the inequality constraints
b=[b; b_tmp];

% *************************************************
% Call the solver
[x, fval] = linprog(f,A,b,Aeq,beq,lb,ub);
Pgrid(1:time_intervals); % Positive values denote power absorption, negative values denote power injection
Petor=(time_intervals+1:2*time_intervals); % Positive values denote charging power, negative values denote discharging power
Total_cost_of_electricity=fval;

% Calculate the state of charge of the storage in each interval
battery_capacity=zeros(1, time_intervals);
battery_capacity(1)=IC+1/4*Petor(1);
for j=2:time_intervals
    battery_capacity(j)=battery_capacity(j-1)+1/4*Petor(j);
end```

Results:

1) Case 1: Solution of the proposed optimization problem

- In Figure (a) positive values indicate power absorption and negative values indicate power injection into the grid. Similarly, in Figure (b) positive values indicate charging and negative values indicate discharging of the storage.
- In order to minimize the cost of electricity, power is absorbed from the grid during low cost periods to charge the storage and to satisfy the load demand. The storage is discharged during high cost periods to satisfy the load demand and the extra power is injected to the power grid.
- The optimization algorithm has found the minimum daily cost of electricity based on the objective function. However, in terms of the system operation the solution is not acceptable. For example, between 22 pm and 24 pm, the storage is charged from the power grid and then the stored energy is injected back to the power grid. However, the cost of electricity is the same between 22 pm and 24 pm and so there is no profit from this action (the same cost is incurred if the storage is not utilized). Therefore, there are multiple solutions of this optimization problem. In the perspective of the system operation we seek the solution that does not utilize the storage if there is no profit in order to increase the expected lifetime of the storage. As a result, the proposed problem formulation must be revised.
2) Case 2: It is assumed that the storage does not exist.

Daily cost of electricity= 4.29 €

Note: In order to find the cost of electricity in the case in which the storage is not present is sufficient to set $\overline{SOC} = 0$ kWh in the optimization problem.

**Homework:** Maximum flow through a network.

When a network has capacity limitations on the flow through arcs there is often interest in finding the maximum flow of some commodity between sources and sinks. The objective is to maximize the flows into the sources and out of the sinks while each arc has an upper capacity. Use linear programming to solve the problem.

Note that is more efficient to use a specialized algorithm for this type of problem: this is the so-called Ford and Fulkerson algorithm.