ECE801
Monitoring and Estimation

Introduction

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Outline

- Topics of the course
- Motivation
- Modeling Overview
Motivation - Monitoring of Critical Infrastructures

- Intelligent Transportation Systems
  - Traffic monitoring
  - Autonomous vehicles
- Power Systems
- Water networks
- Telecommunication networks
Transportation (Autonomous Vehicles)

\[ \dot{x} = v = \frac{dx}{dt} \]

State Update Equation

\[ x_{i+1} = x_i + \Delta t \dot{x}_i \]

State Extrapolation Equation
Transportation (Autonomous Vehicles)
Power Networks

SCADA Measurements

Physical laws

State Estimation

\[
\begin{bmatrix}
P_{flow_{ij}} \\
Q_{flow_{ij}} \\
P_{inj_i} \\
Q_{inj_i} \\
|V_i|
\end{bmatrix}
\]

\[
\sum_{i} I_i = 0
\]

[|V_1|, \hat{\theta}_1, |V_2|, \hat{\theta}_2, ...]

Voltage Magnitudes & Angles

Input

Output

1

2

3

1

2

3

○ Active/reactive power flow measurement

□ Active/reactive injection measurement

△ Voltage magnitude measurement

Active/reactive power flow measurement

Active/reactive injection measurement

Voltage magnitude measurement

Complete & consistent network representation

Active/reactive power flow measurement

Active/reactive injection measurement

Voltage magnitude measurement
Water Networks

- Reservoir
- Pipe
- Flow direction
- Demand Node
- Pump
- Valve
- Leakage
Modeling

- **Model**
  - It is a set of equations or a piece of software (simulator) that imitates the behavior of the real system.
  - There may be several models that can capture the behavior of a system.

- **Modeling is mostly an art and not an exact science.**
  - Depending on the answers we are looking for, models can be very detailed and complex or they can be very simple.
A model predicts what the system’s output would be given an input $u(t)$.

A model is as good as its input: garbage in, garbage out!
Concept of State

Suppose that at a time instant $t_1$, $u(t_1)=a$ and $y(t_1)=Y$. Then, at time $t_2$, $u(t_2)=a$ then what is $y(t_2)=?$.

Example: Let

$x(t)=x(t-1)+u(t)$

$y(u(t))=u(t)+x(t)+5$

The state of a system at time $t_0$ is the information required at $t_0$ such that the output $y(t)$, for all $t\geq t_0$, is uniquely determined from this information and from the input $u(t)$, $t\geq t_0$. 
State Space Modeling

- **State equations**: The set of equations required to specify the state $x(t)$ for all $t \geq t_0$ given $x(t_0)$ and the function $u(t)$.

- **State Space $X$**: The set of all possible values that the state can take.

- **Examples**:

  \[
  \dot{x}(t) = f(x(t), u(t), t) \quad \quad x_{k+1} = f(x_k, \ldots, x_0, u_k, \ldots, u_0, k)
  \]

  \[
  y(t) = g(x(t), u(t), t) \quad \quad y_k = g(x_k, \ldots, x_0, u_k, \ldots, u_0, k)
  \]

  \[
  x(t_0) = x_0
  \]
System Classification

Continuous State – Continuous Time

Discrete State – Continuous Time

Continuous State – Discrete Time

Discrete State – Discrete Time
In many occasions the input functions $u(t)$ are not known exactly but we can only characterize them through some probability distribution.

- Signal noise at a mobile receiver
- Arrival time of customers at a bank
- ...

If the input function is not known exactly, then the state cannot be determined exactly, but it constitutes a random variable.

A system is stochastic if at least one of its output variables is a random variable. Otherwise the system is deterministic.

In general, the state of a stochastic system defines a random process.
Parameter vs State Estimation

- Parameter estimation refers to the estimation of the value of a constant parameter of our model (e.g. the resistance of a resistor)

- State estimation refers to estimating the state of a variable as it changes over time.
Parameter estimation example

- The true value of a resistor’s resistance is not known exactly. Thus we measure it with two multimeters. The obtained measurements are
  \[
  y_1 = x + v_1, \quad \text{and} \\
  y_2 = x + v_2
  \]

Where \( x \) is the resistance we are looking for and \( v_1 \) and \( v_2 \) are the noise uncertainty of the instrument (assumed additive). Both are assumed Gaussian with 0 mean and variance \( \sigma_1^2 \) and \( \sigma_2^2 \) respectively.

**Question**: what is your best estimate for the value of \( x \)?
Parameter estimation example

- If $y_1 = 25$ and $\sigma_1 = 10$, then $\hat{x} = y_1 = 25$
- If $y_2 = 20$ and $\sigma_2 = 5$, then maybe $\hat{x} = y_2 = 20$
- Can we do better?
Parameter estimation example

- **Average:** $\hat{x} = \frac{y_1 + y_2}{2} = 22.5$
- **What would be the variance in this case?**

$$
\sigma^2 = \frac{\sigma_1^2 + \sigma_2^2}{4} = \frac{100 + 25}{4} = 31.25 \quad \Rightarrow \sigma = \sqrt{31.25} = 5.6
$$
Parameter estimation example

- Weighted Average: \( \hat{x} = w_1 y_1 + w_2 y_2 \)
- How can we find the weights \( w_1 \) and \( w_2 \)?
- We would like to have \( E[\hat{x} - x] = 0 \)

\[
E[\hat{x} - x] = E[w_1 y_1 + w_2 y_2 - x] = \\
= E[w_1(x + v_1) + w_2(x + v_2) - x] = 0 \\
\Rightarrow w_1 x + w_2 x - x = 0 \quad \Rightarrow w_1 = 1 - w_2
\]

- So, lets assume \( w_1 = w \) and \( w_2 = 1 - w \)
- We need to find \( w \) that minimizes the estimator variance

\[
J = E[(\hat{x} - x)^2] = E[(w y_1 + (1 - w) y_2 - x)^2]
\]
Parameter estimation example

- So, what is \( w \)?

\[
J = E[(\hat{x} - x)^2] = E[(wy_1 + (1 - w)y_2 - x)^2] \\
= E[(w(x + v_1) + (1 - w)(x + v_2) - x)^2] \\
= E[(wv_1 + (1 - w)v_2)^2] \\
= w^2\sigma_1^2 + (1 - w)^2\sigma_2^2
\]

- What \( w \) minimizes \( J \)?

\[
\frac{dJ}{dw} = 2w\sigma_1^2 - 2(1 - w)\sigma_2^2 = 0
\]

- Therefore

\[
w_1 = w = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad \text{and} \quad w_2 = 1 - w = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}
\]
Parameter estimation example

- **Weights**
  
  \[ w_1 = w = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \frac{25}{125} = 0.2 \quad w_2 = 1 - w = 0.8 \]

- **Weighted Average:** \( \hat{x} = 0.2y_1 + 0.8y_2 = 21 \)

- **With variance** \( \sigma^2 = 0.2^2\sigma_1^2 + 0.8^2\sigma_2^2 = 20 \), so \( \sigma = \sqrt{20} = 4.5 \)